Rate Optimization in Multi-Antenna Relay-Assisted Cognitive Radio Networks

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***Abstract*—This paper presents the performance analysis and rate optimization of multi-antenna relay-assisted cognitive radio networks. The proposed model consists of a pair of primary users (PUs), a number of pairs of secondary users (SUs), and a relay station equipped with multiple antennas. In this cognitive radio network(CRN), each pair of SUs communicates via the relay station for longer range communications. The relay station employs physical layer network coding to precode the incoming messages and forward them to all SUs without causing harmful interference to primary user communications. The weighted sum rate is used as the objective criterion. A novel beamforming scheme is proposed for the SUs and the relay station. The proposed scheme optimizes the transmission rates between the SUs and the relay station while suppressing the interference to an acceptable level at the PUs. Simulations are carried out in two scenarios related to SUs: one is with channel state information (CSI) and the otheris without CSI. It is shown that the rate of the cognitive radio network increases with the number of antennas at the SU nodes and the relay station.**

I. Introduction

Cognitive radio (CR) has shown the potential to overcome spectral limitations and improve spectral utilization efficiency in wireless communication networks. Secondary users (SUs) can monitor the given environment in real time and change their transmission or receiption parameters to communicate effectively without incurring harmful interference to primary users (PUs). Several schemes have been studied to increase the wireless communication range, such as cooperative communication techniques, distributed beamforming, and relay-assisted communication.

Relay channel modeling can aid the bidirectional signal transmission between users. Network coding is an effective technique to improve the linkage and data rate [1]. The conventional amplify-and-forward scheme linearly aggregates the incoming signals, so the quality of the received signals cannot be ensured. When equipped with multiple antennas, the amplify-and-forward scheme shows beamforming characteristics, which lead to higher data transmissions. The weighted sum rate (WSR) [2][3] is used as a metric to measure the performance of the proposed scheme in different scenarios.

In [4], the cognitive base station offers to help the PU transmit the data. As a reward, the CR system will be able to share the spectrum with the primary system. In [5], overlay CR channels were studied. In [6], due to that time division application, SUs sent signals in the idle time slots of the PUs. In the CRN discussed herein, PUs and SUs transmit data simultaneously because the multi-antenna system makes full use of the space domain.

In this proposed cognitive radio network (CRN), we consider the multi-way relay scenario, similar to [7] [8], where multiple users exchange data via a single relay. The application of multiple antennas at the relay station and SUs provides extra spatial degrees of freedom that can significantly boost throughput.

Multi-way relay is studied in this CRN, whereby the relay station and all SUs are equipped with multiple antennas. To maximize the rate of the relay station and the SUs, two scenarios are considered.

- Because of the unavailable channel state information (CSI) of the PUs, the SUs transmit signals to all directions without causing serious influence to PUs in the uplink channel.
- We propose to combine zero forcing(ZF) and an eigenvalue algorithm to send signals at the null space of the PUs, which suppresses interference from the CRN to the PUs and improves the rate at the relay station.

The block diagonalization(BD) is applied in the downlink channel. This method can be limited by the number of antennas at the relay station. When the number of antennas become large enough, BD is easy to realize, because massive multiple input, multiple output (MIMO) technology has been incorporated in [9][10]. Our simulation results validate the proposed multi-way relay scheme and show the throughput improvement.

The following notational convention is used throughout this paper: scalars are in normal fonts, boldface lowercase letters denote vectors, and boldface uppercase letters denote matrices. For any matrix $\mathbf{X}$, $tr(\mathbf{X})$ is the trace of $\mathbf{X}$, $\det(\mathbf{X})$ is the determinant of $\mathbf{X}$, and $||\cdot||$ is the Euclidean norm.

The remainder of this paper is organized as follows: Section II gives a general description of the system model. The problem formulation is introduced in Section III. The eigenvalue algorithm and block diagonalization method are discussed in Section IV. The simulation results are presented in Section V. The conclusion and future work are given in Section VI.

II. System Model

The proposed system model deals with a cognitive radio network sharing the spectrum resource with a primary network, as shown in Fig. 1. In the primary network, it is assumed that there is a pair of PUs. In the cognitive radio network, there are K pairs of SUs, and one relay station assisting in extending the communication ranges of the SUs.

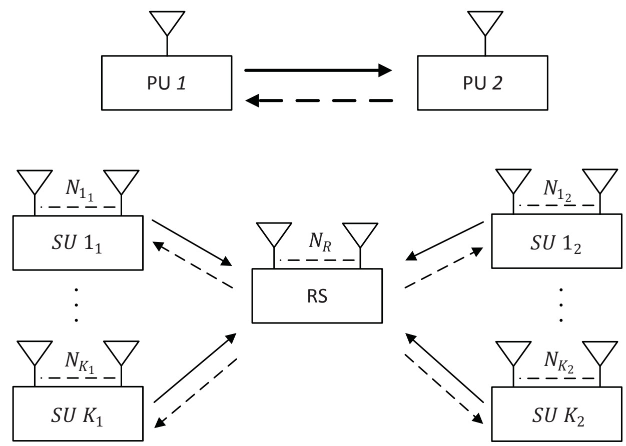

Fig. 1. System model. There is a pair of primary users equipped with one antenna, K pairs of secondary users and a cognitive radio relay station

At the relay station, we use the two-phase transmission protocol with equal time slot. In Fig. 1, the solid lines represent the first phase transmission, and the dashed lines represent the second phase transmissions. The primary network is a point to point communication. Assume that the PUs only communicate with each other and each PU is equipped with a single antenna. In the first phase, PU 1 sends signals to PU 2. In the second phase, PU 2 sends signals to PU 1. While the rate of the primary network can be guaranteed, the cognitive network can opportunistically use the frequency of the primary network. But the interferences resulting from the secondary users' activities must satisfy the constrain to the minimum throughput of the primary network.

The CRN consists of K pairs of secondary users and one relay station. The total number of the secondary users is assumed to be $2K$. The SU k_i (where $k \in 1, 2, \cdots, K, i \in 1, 2$) represents one of the kth pair users, which is equipped with N_{k_i} antennas and the relay station is equipped with N_R antennas. Assume there are no direct links between SUs due to the distance and each pair communicates via the relay station. In the first phase, all SUs access the channels and send signals to the relay station at the same time. The relay station receives the signal from the SUs and then uses the physical-layer network coding to combine the received signal y_R and broadcast the signal to all SUs. Without loss of generality, the transmission media are assumed to be Gaussian radio channels. The exact relaying schemes will be described in the next section.

As discussed above, in the first phase, PU 1 sends signals to PU 2; and all of the secondary users transmit signals to the relay station at the same time. In the second phase, PU 2 sends signals to PU 1; and the relay station broadcasts signals to all of the SUs simultaneously. Thus interferences exist between the primary system and the CRN.

We consider two channel state scenarios. One deals with the CSI of the PUs that is not available at the SUs. The other is that the CSI of the PUs is known at the SUs. In the first scenario, there are no communications between the primary users and the CRN; but the SUs in the CRN can estimate the path loss to the PUs and then adapt the transmitting powers to guarantee the communication quality of the primary system. However, beamforming cannot be used in this case, because SUs do not know the direction of the PUs. In the second situation, because SUs have knowledge of the CSI of the PUs, all SUs are expected to explore the beamforming matrices to mitigate the interference to the PUs. In both scenarios, we assume that the CSI of the PUs is available to the relay station, so that the relay station can broadcast signals in the null space of the PU's channel.

III. Problem Formulation

In the system model shown in Fig. 1, the first phase is operated as follows.

In the primary network, PU 1 sends the signal s_{p_1} with a unit of normalized power to PU 2 , such that $s_{p_1} s_{p_1}^* = 1$. The signal received y_{p2} at the PU 2 can be expressed as

$$y_{p_2} = h_{p_{12}} s_{p_1} + \sum_{i=1}^{n} \mathbf{h}_{k_i p_2} \mathbf{W}_{k_i} \mathbf{s}_{k_i} + n_{p_2} \tag{1}$$

where the $h_{p_{12}} \in \mathcal{C}$ represents the transmission channel in the first phase. The $\mathbf{h}_{k_i p_2} \in \mathcal{C}^{1 \times N_{k_i}}$ represents the channel between the SU k_i and the PU 2. The summation term in (1) is the additive interference from the SUs. The scalar n_{p2} is the gaussian noise at the PU 2 with a probability density function (pdf) $\mathcal{CN}(0, \sigma_{p_2}^2)$, where $\sigma_{p_2}^2$ is the noise variance at the PU 2.

In the CRN, SU k_i transmits signal s_{k_i} with one data stream and the covariance is unit one. Before transmitting to the relay station, an $N_{k_i} \times 1$ beamforming vector $\mathbf{w}_{k_i}$ multiplies the transmitted signal. Thus the transmitted signal at the SU k_i is $\mathbf{x}_{k_i} = \mathbf{w}_{k_i} s_{k_i}$. The covariance of $\mathbf{x}_{k_i}$, represented by $\rho_{k_i}^2$, is the transmitting power at SU k_i. We use the matrix $\mathbf{H}_{k_i} \in \mathcal{C}^{N_R \times N_{k_i}}$ to represent the uplink channel between the relay station and the SU k_i. The signal received vector $\mathbf{y}_R$ at the cognitive relay station is given:

$$\mathbf{y}_R = \sum_{k=1}^{K} \sum_{i=1}^{2} \mathbf{H}_{k_i} \mathbf{w}_{k_i} s_{k_i} + \mathbf{h}_p s_{p_1} + \mathbf{n}_R \tag{2}$$

The second term $\mathbf{h}_p s_{p_1}$ in (2) is the interference from the PU 1, where the $\mathbf{h}_p \in \mathcal{C}^{N_R \times 1}$ is a vector which represents the channel between the relay station and PU 1. $\mathbf{n}_R$ is the noise vector at the relay station with independent and identically distributed(i.i.d) samples following the zero mean circularly symmetric complex Gaussian (CSCG) distribution, denoted by $\mathcal{CN}(0, \sigma_R^2 \mathbf{I}_{N_R})$, where σ_R^2 is the noise covariance at the relay.

In the second phase, PU 2 transmits signals to PU 1 and the relay station broadcasts the signals simultaneously to all SUs. The PU 2 transmits signal s_{p_2} to PU 1. This process is similar to the first phase. The received signal y_{p_1} at the PU 1 is:

$$y_{p_1} = h_{p_{21}} s_{p_2} + \mathbf{h}_{Rp_1} \mathbf{x}_R + n_{p_1} \tag{3}$$

where $h_{p_{21}} \in \mathcal{C}$ represents the radio channel from PU 2 to PU 1 in the second phase. The channel coefficients between the relay station and the PU 1 are $\mathbf{h}_{Rp_1} \in \mathcal{C}^{1 \times N_R}$. The scalar n_{p_1} is the noise at the PU 1 with pdf $\mathcal{CN}(0, \sigma_{p_1}^2)$, where $\sigma_{p_1}^2$ is the noise covariance.

The relay station uses physical layer network coding to linearly combine the received signal $\mathbf{y}_R$ with the $N_R \times N_R$ beamforming matrix $\mathbf{W}_R$ and then broadcasts the signal to all SUs. The transmitted power ρ_R^2 at the relay station is the covariance of $\mathbf{W}_R$. The forwarded signal $\mathbf{x}_R$ is:

$$\mathbf{x}_R = \mathbf{W}_R \frac{\mathbf{y}_R}{||\mathbf{y}_R||} \tag{4}$$

In the second phase, $\mathbf{y}_{k_j}$ is the received signal at the SU k_j ($j \in 1, 2, j \neq i$). After subtracting self-interferences by using network coding protocal, $\mathbf{y}_{k_j}$ is expressed as:

$$\begin{aligned} \mathbf{y}_{k_j} &= \mathbf{G}_{k_j} \mathbf{x}_R + \mathbf{h}_{\mathbf{p_2 j}} s_{p_2} + \mathbf{n}_{k_j} \\ &= \mathbf{G}_{k_j} \mathbf{W}_R \mathbf{H}_{k_i} \mathbf{w}_{k_i} s_{k_i} \\ &+ \sum_{m \neq k} \sum_{i=1,2} \mathbf{G}_{k_j} \mathbf{W}_R \mathbf{H}_{m_i} \mathbf{w}_{m_i} s_{m_i} \\ &+ \mathbf{G}_{k_j} \mathbf{W}_R \mathbf{h}_p s_{p_1} + \mathbf{h}_{\mathbf{p_2 j}} s_{p_2} \\ &+ \mathbf{G}_{k_j} \mathbf{W}_R \mathbf{n}_R + \mathbf{n}_{k_j} \end{aligned} \tag{5}$$

The first term in this expression represents the desired signal from the SU k_i, the second term is the interference from the other pairs of SUs, and the third and fourth terms in the formula are the interference from the PUs. The last term is the noise vector at the SU j with i.i.d CSCG distribution, denoted by $\mathcal{CN}(0, \sigma_{k_j}^2 \mathbf{I}_{N_{k_j}})$, where $\sigma_{k_j}^2$ is the noise covariance at the SU k_j. In Equation (5), $\mathbf{G}_{k_j} \in \mathcal{C}^{N_{k_j} \times N_R}$ is assumed to be the downlink matrix between the SU k_j and the relay station.

Based on the transmit protocol described above, the rate at the primary system and the cognitive radio network can be elaborated as follows.

Assume R_{p_1} and R_{p_2} are the rates at the PU 1 and PU 2, respectively, such that [11]:

$$R_{p_1} = \frac{1}{2} \log(1 + \frac{h_{p_{21}} h_{p_{21}}^*}{||\mathbf{h}_{Rp_1} \mathbf{x}_R||^2 + \sigma_{p_1}^2}) \tag{6}$$

and

$$R_{p_2} = \frac{1}{2} \log(1 + \frac{h_{p_{12}} h_{p_{12}}^*}{\sum_{i=1}^{n} ||\mathbf{h}_{ip_2} \mathbf{w}_i||^2 + \sigma_{p_2}^2}) \tag{7}$$

The rate at the relay station is:

$$\begin{aligned} R_r =& \frac{1}{2} \log\det(\mathbf{I_{N_R}} + \sum_{k=1}^{K} \sum_{i=1}^{2} \mathbf{w}_{k_i}^H \mathbf{H}_{k_i}^H \\ & \times (\mathbf{h}_p \mathbf{h}_p^H + \sigma_R^2 I_{N_R})^{-1} \times \mathbf{H}_{k_i} \mathbf{w}_{k_i}) \end{aligned} \tag{8}$$

The rate at the SU k_j is given as:

$$\begin{aligned} R_{k_j} =& \frac{1}{2} \log\det(\mathbf{I}_{N_{k_j}} + \mathbf{G}_{k_j} \mathbf{W}_R \mathbf{H}_{k_i} \mathbf{w}_{k_i} \\ & \times (\sum_{m \neq k} \sum_{i=1,2} \mathbf{G}_{k_j} \mathbf{W}_R \mathbf{H}_{m_i} \mathbf{w}_{m_i} \mathbf{w}_{m_i}^H \mathbf{H}_{m_i}^H \mathbf{W}_R^H \mathbf{G}_{k_j}^H \\ & + \mathbf{G}_j \mathbf{W}_R \mathbf{h}_p \mathbf{h}_p^H \mathbf{W}_R^H \mathbf{G}_j^H + \mathbf{h}_{\mathbf{p_2 j}} \mathbf{h}_{\mathbf{p_2 j}}{}^H \\ & + \sigma_R^2 \mathbf{G}_{k_j} \mathbf{W}_R \mathbf{W}_R^H \mathbf{G}_{k_j}^H + \sigma_j^2 \mathbf{I}_{\mathbf{N_{k_j}}})^{-1} \\ & \times \mathbf{w}_{k_i}^H \mathbf{H}_{k_i}^H \mathbf{W}_R^H \mathbf{G}_{k_j}^H) \end{aligned} \tag{9}$$

In the CRN, we must satisfy the communication quality of the primary network. Under this necessary condition, the rate optimization is carried out in the CRN. So the objective function is formulated as:

$$\max \ R = \sum_{k=1}^{K} \sum_{j=1}^{2} m_{k_j} R_{k_j} \tag{10}$$

s.t.

$$\min(R_{p_1}, R_{p_2}) \geq \gamma \tag{11}$$

$$tr(\mathbf{x}_{k_i} \mathbf{x}_{k_i}^H) \leq P_{k_i} \tag{12}$$

$$tr(\mathbf{x}_R \mathbf{x}_R^H) \leq P_m \tag{13}$$

where P_{k_i} is the maximum power level at SU K_i, P_m is the power constraint at the relay station, and γ is the rate threshold in the primary system, and it is less than the rate at the PUs without any interferences.

IV. Case Study and Anaysis

In this section, two scenarios will be discussed based on whether the CSI of the PUs is available at the SUs. Given the CSI, the SUs can transmit signals to PUs at the null space of the channel. Under this situation, the interference at the PUs from the CRN will be suppressed but at the cost of decreasing the power efficiency, because only the signals transmitted in the null space are useful. If the transmission power is large enough, the power loss can be ignored; and the relay station can achieve a high rate. In this section, we assume the relay station always knows the CSI of the PUs by utilizing pilot signals, and maintain synchronization between the primary network and the CRN.

A. The uplink channel

1) Without CSI of PU at the SUs: In the system model, it is assumed that the communication happens in a slow fading channel. The SUs can evaluate the pathloss of PUs from the previous phase. Let's assume that f_{k_i} is the path loss between PU 2 and the SU k_i.

The corresponding interference power I at the PU 2 can be obtained from (7) with the threshold γ, that is:

$$I = \frac{h_{p_{12}} h_{p_{12}}^*}{2^{2\gamma} - 1} - \sigma_{p_2}^2 \tag{14}$$

Thus, the constraint (11) translates to the total transmitting power of the secondary users P_s:

$$P_s = \sum_{k=1}^{K}\sum_{i=1}^{2} \rho_{k_i} \leq I + \sum_{k=1}^{K}\sum_{i=1}^{2} f_{k_i} \tag{15}$$

In order to keep the fairness of each SU, let the interference from each SU at PU 2 be equal. So ρ_{k_i} can be expressed as:

$$\rho_{k_i} \leq \frac{P_s}{2K} \tag{16}$$

The maximum rate at the relay station will be obtained when the rate of PU 2 satisfies the threshold requirement. Given the threshold γ, the corresponding interference I can be obtained from equation (14), and the transmitting power p can be obtained from Equations (15) and (16). Suppose there exists $\delta > 0$, then $\gamma' = \gamma + \delta > \gamma$ and the corresponding interference of γ' is I' with the transmit power p' at SUs. Obviously, $I' < I$. So $p' < p$, the transmitted power at the SUs must decrease to satisfy the constraint Equation (11). It can be seen from Equation (8), that the rate at the relay station will decrease. At the threshold, the relay station can get the maximum rate. In other words, if the transmission power at the SUs can guarantee the threshold at the PUs, all data rate at the PUs can also be guaranteed. So the constraint in Equation (11) can be relaxed to:

$$\min(R_{P_1}, R_{P_2}) = \gamma \tag{17}$$

Under the constraint of Equation (11), the rate at the relay station increases as the transmitting powers at the SUs increase. From Equations (12), (16) and (17), the transmit power at SU k_i is relaxed to:

$$\rho_{k_i} = \min(\frac{P_s}{2K}, P_{k_i}) \tag{18}$$

If beamforming techniques are applied at the SUs, the power can concentrate at some directions rather than all directions. Because the SUs do not know the position of PU 2, beamforming techniques cannot be applied at the SUs.

2) With CSI of PUs at the SUs: In this case, PU informs some information to the relay station and the SUs. Knowing the CSI of PUs, the SUs can send messages in the null space of the interference channel. Thus the constraint of Equation (11) can be ignored. When the SNR is high, this method can achieve a desired rate at the SUs.

In this case, the beamforming vector $\mathbf{w}_{k_i}$ at SU k_i is designed based on the zero forcing (ZF) method in [12]. As mentioned previously, $\mathbf{H}_{k_i}$ is the channel of SU k_i to the relay station. Its projection at the space of $\mathbf{h}_{k_ip_2}$ is $\frac{\mathbf{H}_{k_i}\mathbf{h}_{k_ip_2}}{||\mathbf{h}_{k_ip_2}||^2}\mathbf{h}_{k_ip_2}$, and the channel $\mathbf{H}_{k_i}$ projects into the null space of $\mathbf{h}_{k_ip_2}$ is:

$$\mathbf{H}_{k_i\perp} = \mathbf{H}_{k_i} - \frac{\mathbf{H}_{k_i}\mathbf{h}_{k_ip_2}}{||\mathbf{h}_{k_ip_2}||^2}\mathbf{h}_{k_ip_2} \tag{19}$$

Eigenvalue algorithm can be used to solve this problem. The square matrix $\mathbf{H}_{k_i\perp}^H\mathbf{H}_{k_i\perp}$ represents a direction in a multi-dimensional space. Its eigenvectors are normalized and lie in the same direction as the coordinate axis of this space. The eigenvalues are the corresponding length of the projection to the direction of the eigenvectors.

Because the product of $\mathbf{H}_{k_i\perp}^H\mathbf{H}_{k_i\perp}$ is a hermite matrix, the eigenvector and the eigenvalue can be calculated as follows:

$$\mathbf{H}_{k_i\perp}^H\mathbf{H}_{k_i\perp}\mathbf{v}_{k_i\perp} = \lambda_{k_i\perp}\mathbf{v}_{k_i\perp} \tag{20}$$

Here we use $\lambda_{k_i\perp}, \mathbf{v}_{k_i\perp}$ for the biggest eigenvalue and its corresponding eigenvector. And it can be shown $\mathbf{h}_{k_ip_2}\mathbf{v}_{k_i\perp} = 0$.

Proof: Left multiplying of equation (20) by $\mathbf{h}_{k_ip_2}$:

$$\mathbf{h}_{k_ip_2}\mathbf{H}_{k_i\perp}^H\mathbf{H}_{k_i\perp}\mathbf{v}_{k_i\perp} = \mathbf{h}_{k_ip_2}\lambda_{k_i\perp}\mathbf{v}_{k_i\perp} \tag{21}$$

Because $\mathbf{H}_{k_i\perp}$ is orthognal to $\mathbf{h}_{k_ip_2}$, we have $\mathbf{H}_{k_i\perp}\mathbf{h}_{k_ip_2} = \mathbf{0}$. The left-hand-side of (21) equals to zero, therefore:

$$\mathbf{h}_{k_ip_2}\mathbf{v}_{k_i\perp} = \mathbf{0} \tag{22}$$

■

Then, the data stream selects the eigenvector $\mathbf{v}_{k_i\perp}$ corresponding to the largest eigenvalue $\lambda_{k_i\perp}$ as its beamforming matrix. So:

$$\mathbf{w}_{k_i} = \rho_{k_i}\mathbf{v}_{k_i\perp} \tag{23}$$

Where ρ_{k_i} is added to satisfy the constraint (12).

Because $\mathbf{w}_{k_i}^H\mathbf{w}_{k_i} = \rho_{k_i}^2$, where $\rho_{k_i}^2$ is the transmit variance at the SU k_i, it should satisfy the constraint (12).

$$\mathbf{w}_{k_i}^H\mathbf{H}_{k_i}^H\mathbf{H}_{k_i}\mathbf{w}_{k_i} = \rho_{k_i}^2\lambda_{k_i\perp} \tag{24}$$

By using this precoding method, the rate at the relay station becomes:

$$R_r = \frac{1}{2}\log(1 + \sum_{k=1}^{K}\sum_{i=1}^{2}\frac{\rho_{k_i}^2\lambda_{k_i\perp}}{h_p h_p^H + \sigma_R^2}) \tag{25}$$

Because the signals from the SUs only transmitted in the null space of the channel to PU 2 are effective, some power waste is inevitable. But this algorithm can cancel the interference to the PU 2 effectively. So better performances can be achieved when transmit power is high or the channel is in good condition.

B. The downlink channel

The downlink channel from RS to SUs is a broadcast channel. The physical-layer network coding (PNC) can be used because the SUs can cancel self-interference in the received signals. The relay station precodes the signals received in the first phase with the matrix $\mathbf{W}_R$ and then sends the signals to all SUs. We decompose the matrix $\mathbf{W}_R$ to three parts, such that $\mathbf{W}_R = \mathbf{PDQ}$, where $\mathbf{P} \in \mathcal{C}^{N_R \times L_P}$, $\mathbf{D} \in \mathcal{C}^{L_P \times L_Q}$, $\mathbf{Q} \in \mathcal{C}^{L_Q \times N_R}$. And $\mathbf{PDQ}$ can also be expressed as follows:

$$\begin{aligned} \mathbf{P} &= [\mathbf{P}_1\mathbf{P}_1 \cdots \mathbf{P}_K] \\ \mathbf{D} &= diag\{\mathbf{D}_1\mathbf{D}_2 \cdots \mathbf{D}_K\} \\ \mathbf{Q} &= [\mathbf{Q}_1\mathbf{Q}_2 \cdots \mathbf{Q}_K] \end{aligned} \tag{26}$$

where $\mathbf{P}_k \in \mathcal{C}^{N_R \times L_{P_k}}$, $\mathbf{D}_k \in \mathcal{C}^{L_{P_k} \times L_{Q_k}}$, $\mathbf{Q}_k \in \mathcal{C}^{L_{Q_k} \times N_R}$. Letting $\mathbf{W}_{R_k} = \mathbf{P}_k \mathbf{D}_k \mathbf{Q}_k$, so:

$$\mathbf{W}_R = \sum_{k=1}^{K} \mathbf{W}_{R_k} \tag{27}$$

In the uplink channel, if SUs know the CSI of the PU's channel, eigenvalue algorithm is used, so we can use the $\mathbf{w}_{k_i}$ from Equation (23) to obtain:

$$\mathbf{Q}_{k_i} = \mathbf{w}_{k_i}^H \mathbf{H}_{k_i}^H \tag{28}$$

The completely ZF structure does not account for the fact that the RS should get rid of interpair interference instead of intrapair interference. Because the SUs are equipped with multiple antennas, complete diagonalization of the channel at the relay station is suboptimal, since every SU is able to coordinate the processing of its own receiver signals and cancel the self-interference. In order to maximize the rate at the SU k_j, we have to increase the power of the useful signals or decrease the interference power from other SUs. In other words, to maximize $||\mathbf{G_{k_j}} \mathbf{W}_{R_{k_j}}||^2$ and minimize $||\mathbf{G_j} \mathbf{W}_{R_{m_j}}||^2$ for $(m \neq k)$, the BD technique is used [13], [14].

From the Equation (9), the interference from the primary user and the noise at the relay station are considered to be constraint. Knowing the CSI of primary users, we can project the downlink channel to the null space of the primary user's channel as $\mathbf{G}_{k_i \perp}$.

$$\mathbf{G}_{k_i \perp} = \mathbf{G}_{k_i} - \frac{\mathbf{G}_{k_i} \mathbf{h}_{Rp_1}}{||\mathbf{h}_{Rp_1}||^2} \mathbf{h}_{Rp_1} \tag{29}$$

To find the matrix $\mathbf{P}_R$, such that $\mathbf{G}\mathbf{W}_R$ is a block diagonal matrix. We define:

$$\tilde{\mathbf{G}}_{k\perp} = [\mathbf{G}_{1_1 \perp}, \mathbf{G}_{k-1_2 \perp}, \cdots, \mathbf{G}_{k+1_1 \perp}, \cdots, \mathbf{G}_{K_2 \perp}] \tag{30}$$

To realize $\mathbf{G}_k \mathbf{W}_m = \mathbf{0}(m \neq k)$, we make $\mathbf{W}_k$ lie in the null space of $\tilde{\mathbf{G}}_{k\perp}$. In order to guarantee all SUs satisfy this constraint, $rank(\tilde{\mathbf{G}}_{k\perp}) \leq N_R$. If $N_R \geq \max\{rank(\tilde{\mathbf{G}}_{1\perp}), \cdots, rank(\tilde{\mathbf{G}}_{K\perp})\}$, the operation of block diagonalization will be possible. Thus, it is not required $N_R \geq \sum_{k=1}^{K} \sum_{i=1}^{2} (N_{k_i})$. Given the dimension constraint, let $\tilde{D}_k = rank(\tilde{\mathbf{G}}_{k\perp})$. To decompose $\tilde{\mathbf{G}}_{k\perp}$ with the singular value decomposition (SVD):

$$\tilde{\mathbf{G}}_{k\perp} = \tilde{\mathbf{U}}_k \mathbf{\Sigma}_k [\tilde{\mathbf{V}}_k^{(1)}, \tilde{\mathbf{V}}_k^{(0)}]^H \tag{31}$$

Where $\mathbf{U}_k$ represent the left singular vectors, $\mathbf{\Sigma}_k$ is a diagonal matrix with the singular values as its diagonal element, $\tilde{\mathbf{V}}_k^{(1)}$ holds the first $\tilde{D}_k$ column vectors, and $\tilde{\mathbf{V}}_k^{(0)}$ holds the last $(N_R - \tilde{D}_k)$ column vectors. $\tilde{\mathbf{V}}_k^{(0)}$ is the orthogonal basis for null space of $\mathbf{G}_{k\perp}$.

Then, let L_{P_k} be the rank of $\mathbf{G}_{k_j \perp} \tilde{\mathbf{V}}_k^{(0)}$. In order to perform the BD, $\hat{D}_k \geq 1$ is necessary, because there should be at least one row in $\mathbf{G}_{k\perp}$ that is linearly independent to rows of $\tilde{\mathbf{G}}_{k\perp}$. If channels are highly correlated, the block diagonalization algorithm cannot be used. Assume all users satisfy this constraint, the SVD of $\mathbf{G}_{k_j \perp} \tilde{\mathbf{V}}_k^{(0)}$ can be expressed as:

$$\mathbf{G}_{k_j \perp} \tilde{\mathbf{V}}_k^{(0)} = \mathbf{U}_k \begin{bmatrix} \mathbf{\Sigma}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} [\mathbf{V}_k^{(1)}, \mathbf{V}_k^{(0)}]^H \tag{32}$$

The notation of symbols are the same as in equation (31), $\mathbf{V}_k^{(1)}$ represents the first L_{P_k} right singular vectors. The precoding matrix $\mathbf{P}_{R_k}$ can be expressed as:

$$\mathbf{P}_{R_k} = \mathbf{V}_k^{(1)} \tilde{\mathbf{V}}_k^{(0)} \tag{33}$$

We now have the structure for $\mathbf{D}_k \in \mathcal{C}^{L_{P_k} \times L_{Q_k}}$. Define $d_k = \min(L_{P_k}, L_{Q_k})$:

$$\mathbf{D}_k = \begin{bmatrix} \mathbf{I}_{d_k \times d_k} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{34}$$

Thus, the precoding matrix $\mathbf{W}_{R_k}$ of the kth pair can be obtaied as follows:

$$\mathbf{W}_{R_k} = \rho_R \mathbf{P}_k \mathbf{D}_k \mathbf{Q}_k \tag{35}$$

Where ρ_R should be a constraint to (13).

So the rate at the SU k_j becomes:

$$R_{k_j} = \frac{1}{2} \log \det(I_{N_{k_j}} + \frac{\rho_R^2 \mathbf{G}_{k_j} \mathbf{G}_{k_j}^H}{\mathbf{D}}) \tag{36}$$

Where

$$\begin{aligned} \mathbf{D} = \mathbf{G}_{k_j} \mathbf{W}_R \mathbf{h}_p \mathbf{h}_p^H \mathbf{W}_R^H \mathbf{G}_j^H + \mathbf{h_{p_2 k_j}} \mathbf{h_{p_2 k_j}}^H \\ + \sigma_R^2 \mathbf{G}_{k_j} \mathbf{W}_R \mathbf{W}_R^H \mathbf{G}_{k_j}^H + \sigma_{k_j}^2 \mathbf{I_{N_{k_j}}} \end{aligned} \tag{37}$$

In the CRN, all SUs receive signals with linear Minimize Mean Square Error (MMSE) technique [15]. A matrix $\mathbf{M}_{k_j}$ is used to minimize the error of the received signal at SU k_j. $\hat{\mathbf{y}}_{k_j} = \mathbf{M}_{k_j}^H \mathbf{y}_{k_j}$, such that:

$$\frac{\partial (E(||\hat{\mathbf{y}}_{k_j} - \mathbf{x}_{k_i}||^2))}{\partial \mathbf{M}_{k_j}} = 0 \tag{38}$$

By solving equation (38), we have [15]:

$$\mathbf{M}_{k_j} = (\mathbf{G}_{k_j} \mathbf{G}_{k_j}^H + \mathbf{R}_{n_{k_j}})^{-1} \mathbf{G}_{k_j} \tag{39}$$

Where $\mathbf{R}_{n_{k_j}} = E(\mathbf{n}_{k_j} \mathbf{n}_{k_j}^H)$

Now SUs can extract their desired signals. In the next section, simulation results will show the performance of the system.

V. Simulation Results

In this simulation, we used normalized power. We assumed the maximum transmit powers at SUs were same (i.e. $P_{1_1} = P_{1_2} = P_{2_1} = \cdots = P_{K_2}$). The transmit power at the relay station was $P_m = 2$. The noise powers at the primary system and the cognitive radio network were same i.e.,$\delta_{p_1} = \delta_{p_2} = \delta_R = \delta_j$. The channels between the primary users $h_{p_{12}}, h_{p_{21}}$, and the radio channel among the secondary users and the relay station $\mathbf{H}_{k_i}, \mathbf{G}_{k_i}, (k = 1, \cdots, K, i = 1, 2)$, or from the primary users to the secondary users $\mathbf{h}_{k_i p_2}, \mathbf{h}_{p_{2i}}$ and the relay station $\mathbf{h}_p, \mathbf{h}_{Rp_1}$ were all drawn from the family of random scalars,vectors and matrices with CSCG. Their distributions are set to be $\mathcal{CN}(0,1)$. It is assumed there were two pairs of secondary users in the cognitive radio networks, i.e., $K = 2, n = 4$, and each SU was equipped with two antennas ($N_{k_i} = 2, k = 1,2, i = 1,2$). The RS was assumed to have eight antennas or more, because we want to compare performances with different numbers of antennas at the RS. Both the primary users were equipped with single antennas. The SINR corresponding to the threshold of the rate γ in the primary system was from 0 dB to 20 dB. Each simulation result was obtained from 10^5 random channel realizations.

A. Rate at the relay station without CSI

Fig. 2 shows the relationship between the rate threshold in the primary network and the rate at the RS. It can be found in the figure that as the rate threshold at the PUs increases, the rate at the RS decreases. Because the threshold influences the maximum interference power at PUs, with the maximum transmit powers are affected at SUs. When the threshold at the PU is fixed, the 16 antennas at the RS perform better than the 12 antennas and much better than the 8 antennas. The more antennas at the RS, the higher rate can we achieve. Because the number of antennas at the RS increases, the degree of freedom increases, so does the performance. However the increase in the performance will be smaller as the number of antennas becomes extremely larger. When the number of antennas at the RS increases from 8 to 12, the rate improves about $2bps/Hz$. By increasing the number of antennas at the RS from 12 antennas to 16, it has only a $1bps/Hz$ improvement. Because the number of antennas at the SUs is fixed, the rate will not always increase as the number of antennas at the RS increases.

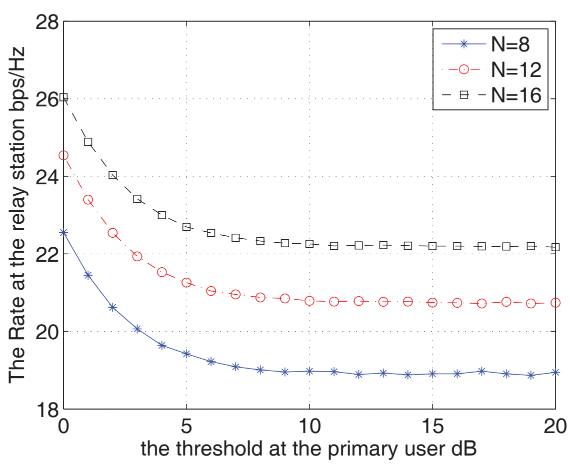

Fig. 2. The rate at the relay station decreases as the rate threshold of the primary system increases, with different receive antenna numbers at the relay station ($N_R = 8, 12, 16,\ K = 4$).

B. Rate at the relay station with CSI

Fig. 3 compares the two scenarios discussed in Section IV-A. When the CSI of the PU is available to the SUs, the transmit power at the SUs is increased as a result of the increase in the rate at the RS. In this scenario, SUs can send signals in the null space of the channel to the PUs, so SUs can transmit signals at any power level as they are not limited by the primary network. If the CSI of the PUs is not available to the SUs, the transmit power increases to a certain value which may cause the rate of the PU to its threshold. In that case, the primary network would not allow the CRN to use the spectrum. Thus, the rate at the RS will not increase as the transmit power increases at SUs, because the SUs have to send signal with the low powers to guarantee the tolerance of the interference to the PUs.

In the low normalized transmitting power lower than 1, the rate at the relay station is not easy to achieve, because the transmit power will be wasted in the space of the PU channels. This is why the rate is lower than the rate without the CSI of the PUs. As the transmit power increases, the performance improves.

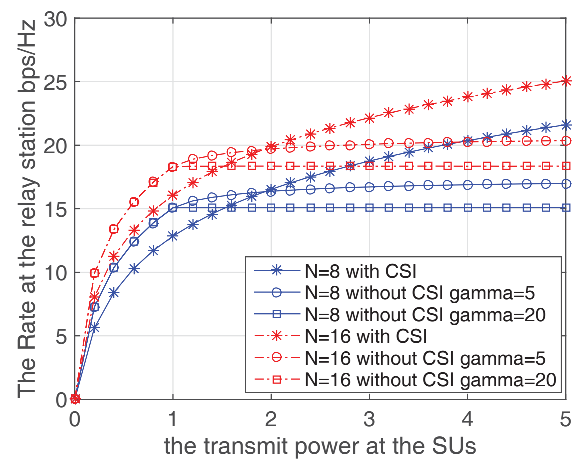

Fig. 3. The transmit power at the SUs increases from 0 to 5. The rate at the RS is obtained under two scenarios. One is the CSI of the PUs available to SUs, the other is without. The receive antennas at the RS is 8 and 16 respectively. The rate threshold at the PUs is 5 bps/Hz or 20 bps/H

C. Block Diagonalization

Fig. 4 illustrates the mean square error at the secondary users using the block diagonalization method described in Section IV-B. As the SNR increases, the channel conditions improve and the MMSE of the signals received at the SUs decreases. If the relay station is equipped with more antennas, ($N = 8, 12, 16$), given an SNR value, the MMSE decreases. However, as the number of antennas at the RS increases, the improvement of the MMSE becomes smaller.

VI. Conclusion

In this paper, we studied a multi way relay in the cognitive radio scenario where all of the secondary users and the relay station are equipped with multiple antennas. In order to maximize the throughput in the cognitive radio network under the constraint of the threshold rate of the primary users, we proposed an eigenvalue algorithm to design the uplink

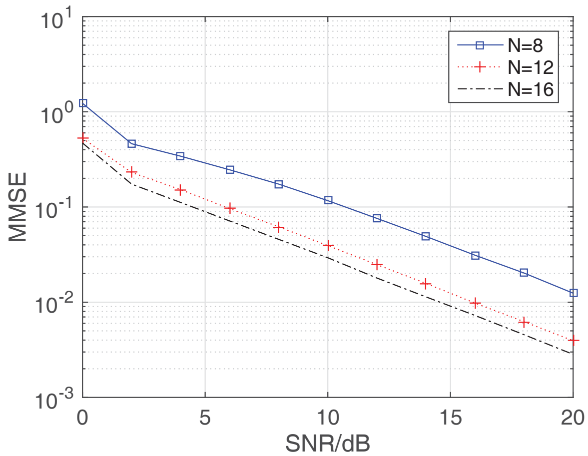

Fig. 4. In the second phase, the mean square error at the secondary users under different SNR and different antennas at the relay station,($N_R = 8, 12, 16$, $K = 4$)

beamforming matrix and the block diagonalization method to deal with the downlink problem. Our simulation results show that if the primary users send some channel information periodically to the cognitive radio network, the interference to the primary users from the cognitive radio network can be suppressed. Thus, the proposed multi-antenna relay assisted cognitive radio can not only guarantee the communication quality of the primary users, but also improve the performance of the cognitive radio network.

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