Two-Stage Comb Decimator with Improved Frequency Characteristic

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***Abstract*—In this paper we consider comb-based decimation structures for high values of decimation factors in which the decimation factor can be presented as a power of x, where x is a prime number. The analysis of the power consumption and the used area for a non-recursive (which also includes the polyphase decomposition), and recursive forms in term of the values of x and the decimation factors, is presented. Based on this analysis, we propose two structures that take the advantages of both: the power efficiency of a non-recursive form, and the area efficiency of a recursive form. The structure, denoted as a *NR-CIC*-1 structure, consists of the first stage in a non-recursive form followed by the second stage in the recursive form. The other structure, denoted as a *NR-CIC*-2 structure, utilizes the polyphase decomposition of the non-recursive form of the first stage, followed by the recursive form in the second stage. The power and area analysis of the proposed structures proves their efficiency. We also present slight modifications for both structures, (*NR-CIC*-3 and *NR-CIC*-4) which do not affect the power and area efficiency, but improve the alias rejections. Two examples are included to illustrate this concept. We also present the analysis of the power consumption and the utilized area in order to show the efficiency of the modified structures. Finally, the VHDL implementation of the proposed structures taking the decimation factor of 512, is presented along with the summary of the area and power consumption.**

***Index Terms*—Analog-to-Digital conversion, sigma-delta modulation, decimation filter, comb filter, CIC structure.**

I. Introduction

OVERSAMPLED Sigma-Delta (ΣΔ) converters have become widely used as a valid alternative to conventional A/D Converters (ADCs) converters. These kind of A/D converters sample the analog input signal with a frequency much larger than the Nyquist frequency, which is expressed through the oversampling ratio (OSR). Although ΣΔ converters were originally conceived for low-frequency, high resolution applications, their use has progressively extended to medium and high-frequency applications [1].

As well known, ΣΔ ADCs are made up of a modulator– where the signal is oversampled and quantization noise shaping takes place – and a decimator, where the oversampled frequency of the modulator is decreased to the Nyquist frequency. Although the modulator is the most critical block, the design of the decimator must be also taken into account in order to optimize the performance of the whole ADC. Indeed, one of the key parts of the decimation stage is the decimation filter, which is responsible for the aliasing rejection introduced in the process of decreasing the sampling rate.

The most popular decimation filter is a comb filter because it requires neither multiplications nor coefficient storage but only additions/subtractions. Due to its simplicity it is usually used in the first stage of decimation. Its transfer function is given as

$$H(z) = \left[\frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}}\right]^K, \tag{1}$$

where M is the decimation factor and K is the number of cascaded filters. Using multirate identity, the implementation of (1) results in the popular Cascaded-Integrator-Comb (CIC) structure proposed by Hogenauer [2], whose block diagram implementation is presented in Fig. 1(a). For large decimation factors, CIC structures present high power consumption due to the integration section, which works at the highest sampling rate and with the full word-length. The alternative is a non-recursive-comb filter with a transfer function given by:

$$H(z) = \left[\frac{1}{M}\sum_{l=0}^{M-1} z^{-l}\right]^K, \tag{2}$$

which has been reported consuming less power [3]-[5]. The most popular non-recursive-comb structure uses a decimation factor that can be represented as a power of two, i.e. $M=2^P$. In this case, it can be shown that the transfer function is given by:

$$H(z) = \frac{1}{2^{PK}}\left[\prod_{i=0}^{P-1}\left(1+z^{-2^i}\right)\right]^K. \tag{3}$$

The above result has been extended in [6] and [7], reporting non-recursive-comb decimation filters which make use of decimation factors that are power of three, $M=3^P$. Indeed, in a more general case, as presented in [8], comb filters can be implemented with non-recursive-comb structures if the decimation factor is a power of any prime number x, i.e. $M=x^P$ where x=2,3,5,7..., including also a combination of them. Fig. 1(b) presents the general implementation diagram of non-recursive-comb with $M=x^P$. In this general case, it can be shown that the transfer function is given by [8]:

$$H(z) = \frac{1}{x^{PK}}\left[\prod_{i=0}^{P-1}\left(1+z^{-x^i}+\cdots z^{-(x-1)x^i}\right)\right]^K; M = x^P \tag{4}$$

An additional advantage of non-recursive-comb filters is that they can be implemented in polyphase form (polyphase-comb), thus moving the filtering to low rate [9]. However,

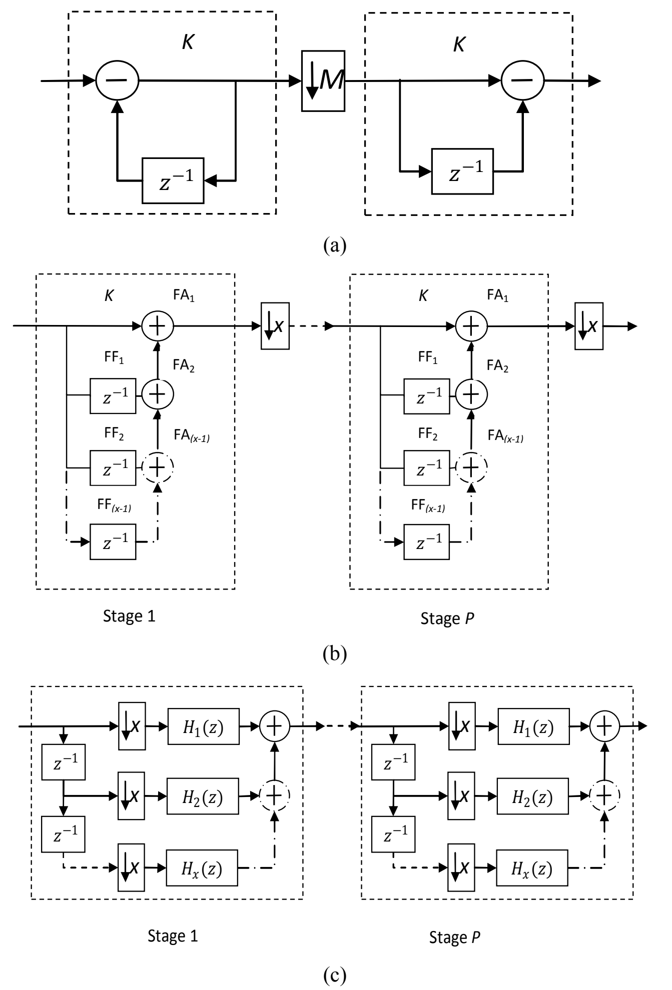


Fig. 1.Diagram implementation of (a) CIC, (b) non-recursive-comb and (c) polyphase-comb structures.

polyphase implementation of non-recursive-comb filters requires multiplier, for the coefficient implementation, which are power and area hungry devices compared with adders. Nevertheless, the coefficients can be implemented by shifts and additions, i.e. only using adders, in order to reduce the power and area requirements [10]. Furthermore, sub-expression sharing techniques can be also used in order to reduce the number of adders [11]-[12]. Fig. 1(c) presents the general block diagram implementation of a polyphase-comb structure where the decimation factor is a power of x. From Fig. 1(c) it can be seen that the sampling frequency of each stage in the polyphase-comb structure is x-times lower than in non-recursive-comb, that´s way polyphase-comb implementations usually requires less power than non-recursive-comb but at the expense of increased area. Therefore, although non-recursive-comb and polyphase-comb structures present less power consumption than CIC topologies, especially at high decimation factors, their required area is relatively high compared with that of the corresponding CIC structure.

Additionally, all comb structures exhibit poor magnitude characteristic. Different methods have been proposed to improve the comb magnitude characteristic [13].

The former analysis shows that the principal issues in the design of comb based decimators, usually treated separately, are:

- Decrease as much as possible the power consumption
- Decrease as much as possible the active area
- Improve magnitude comb characteristic

This paper contributes to this topic and presents two-stage power and area efficient structures where the first stage is in a non-recursive form (with and without the polyphase decomposition) while the second stage is a CIC-based decimation structure. A modified version of the proposed decimator topologies is also presented, demonstrating an improved alias rejection feature while keeping a low power and area configuration efficiency.

As an extension of the work introduced in [14] – which presented comb structures with a low power consumption and area and an improved alias rejection for high decimation factors which are power of two – the goal of this paper is to propose a generalized comb-based structure with a low power consumption, low used area considering high values of decimation factor which can be presented as power of prime number x. In addition, this work proposes to improve the alias rejection in the presented structures, keeping their power and area efficiency.

The paper is organized as follows. Section II discusses the power and area estimations used for the structures under study, namely: CIC, non-recursive-comb and polyphase-comb filters. Section III describes the proposed structures and Section IV analyses how these structures can be improved in terms of alias rejection. The proposed design procedure and some synthesis examples are presented in Section V. Finally, a VHDL implementation in a 0.18µm CMOS technology is shown in Section VI taking M=512 as an example, and conclusions are given in Section VII.

II. Power and Area Estimation of CIC, Non-recursive-comb and Polyphase-comb Structures

The dynamic power consumption of a decimation filter can be estimated by the number of required full adders (FA) and registers (FF) as follows [4],

$$P = \gamma(FA + FF)B_{out}, \quad (5)$$

where γ is the relative sampling frequency of the stage compared with the input sampling frequency, and B_{out} is the word-length increase to avoid overflow. The value of the word-length increase can be calculated as [2]:

$$B_{out} = \lceil B_{in} + K \cdot log_2(M) \rceil, \quad (6)$$

where $\lceil . \rceil$ is the ceiling function and B_{in} is the input word-length.

The used active area, A, can be modeled in a similar way, since this parameter depends also on the number of adders and registers, but it is independent of the sampling frequency, giving[1]:

$$A = (FA + FF)B_{out}. \quad (7)$$

From Fig. 1(a) and using (5)-(7), the power and area estimations for a CIC structure, P_{CIC} and A_{CIC}, can be respectively derived, yielding:

$$P_{CIC} = (FA_I + FF_I)B_{out} + \frac{(FA_C + FF_C)B_{out}}{x^P}, \quad (8)$$

$$A_{CIC} = [(FA_I + FF_I) + (FA_C + FF_C)]B_{out}, \quad (9)$$

where the subscripts I and C denote the integrator and comb sections, respectively.

Since the CIC structure is very regular, the number of full adders and registers can be computed as:

$$FA_I = FA_C = FF_I = FF_C = K \quad (10)$$

Similarly, applying (5)-(7) to the structure of Fig. 1(b), it can be shown that the power and area estimations for the non-recursive-comb structures, $P_{NRC\text{-}x}$ and $A_{NRC\text{-}x}$, are given by:

$$P_{NRC-x} = \sum_{i=1}^{P} \frac{(FA_{NCS} + FF_{NCS}) \cdot \lceil B_{in} + K \cdot log_2(x^i) \rceil}{x^{i-1}}, \quad (11)$$

$$A_{NRC-x} = \sum_{i=1}^{P} (FA_{NCS} + FF_{NCS}) \cdot \lceil B_{in} + K \cdot log_2(x^i) \rceil. \quad (12)$$

where x= 2, 3, 5, 7…, i.e. a prime number, and the subscripts NCS are for one non-recursive-comb stage. Since the non-recursive-comb structure is also very regular, the total number of FA_{NCS} and FF_{NCS}, for each stage, can be determined as:

$$FA_{NCS} = FF_{NCS} = (x - 1)K \quad (13)$$

In the case of the polyphase-comb structure, shown in Fig. 1(c), the same procedure based on (5)-(7) is used to derive the power and area estimation, $P_{PC\text{-}x}$ and $A_{PC\text{-}x}$, giving:

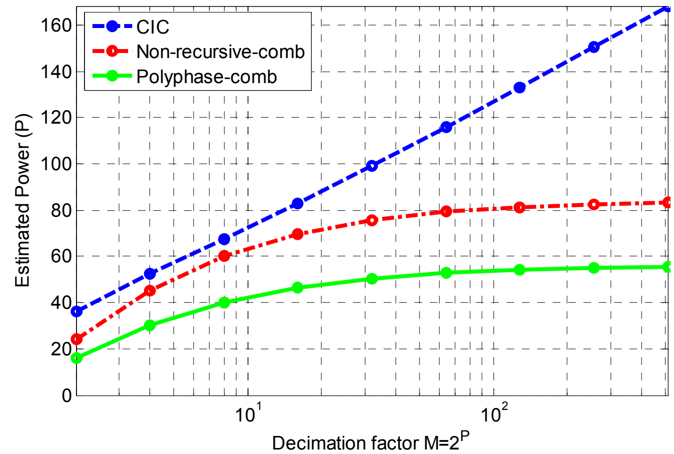
$$P_{PC-x} = \sum_{i=1}^{P} \frac{(FA_{PCS} + FF_{PCS}) \cdot \lceil B_{in} + K \cdot log_2(x^i) \rceil}{x^{i}}, \quad (14)$$

$$A_{PC-x} = \sum_{i=1}^{P} (FA_{PCS} + FF_{PCS}) \cdot \lceil B_{in} + K \cdot log_2(x^i) \rceil, \quad (15)$$

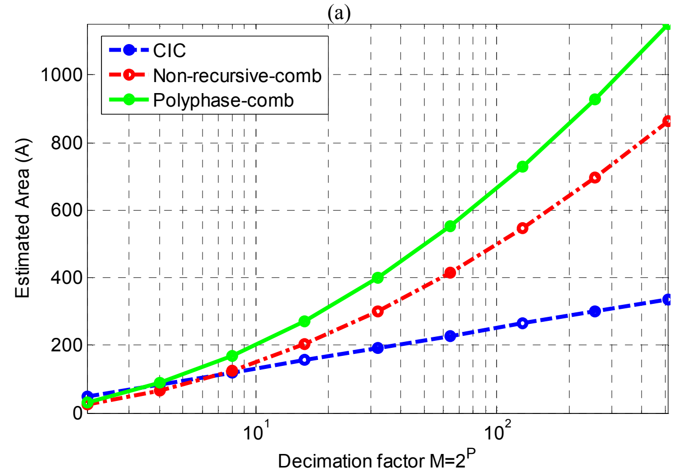
where x is the prime number, and the subscript PCS stands for one polyphase-comb stage. Note that, unlike CIC and non-recursive-comb structures, the number of required FA_{PCS} and FF_{PCS} will depend on the number of cascaded stages K, the coefficient representation and the use or not of sub-expression sharing techniques. As an illustration, Table I shows the number of FA_{PCS} and FF_{PCS} for x= 3 with cascades, K, ranging from 2 to 5, considering binary representation of coefficients and no sub-expression sharing techniques.

This way, by using (8)-(15), and assuming that, for simplicity and comparison proposes, one FA and one FF has the same contribution for power and area, Fig. 2(a) and Fig. 2(b) present the estimated power consumption and the required area, respectively, for CIC, non-recursive-comb and polyphase-comb structures. In Fig. 2 we have considered that the decimation factor is a power of two, i.e. x=2, K=3 and the input word-length is one bit. Note from Fig. 2(a) that, as M increases, the power consumption for CIC grows logarithmically while for non-recursive-comb and polyphase-comb the growth is asymptotic due to the frequency reduction through each stage. The required power in polyphase-comb is lower than in non-recursive-comb due to the lower frequency in each stage. As a result, the power consumption for a CIC filter is higher than that for a non-recursive-comb and polyphase-comb filter, especially for high decimation factors.

On the other hand, from Fig. 2(b) it can be seen that the required area of CIC filter increases logarithmically while in non-recursive-comb and polyphase-comb, the growth is approximately quadratic. As a result, the used area for CIC filter is generally less than that for the corresponding non-recursive-comb and polyphase-comb, especially for high decimation factors. The required area in polyphase-comb is higher than in non-recursive-comb due to the additional number of FA required for the implementation of the coefficients.



(a)



(b)

Fig. 2 Estimated (a) power and (b) area, for CIC, non-recursive-comb and polyphase-comb for M=2^P.

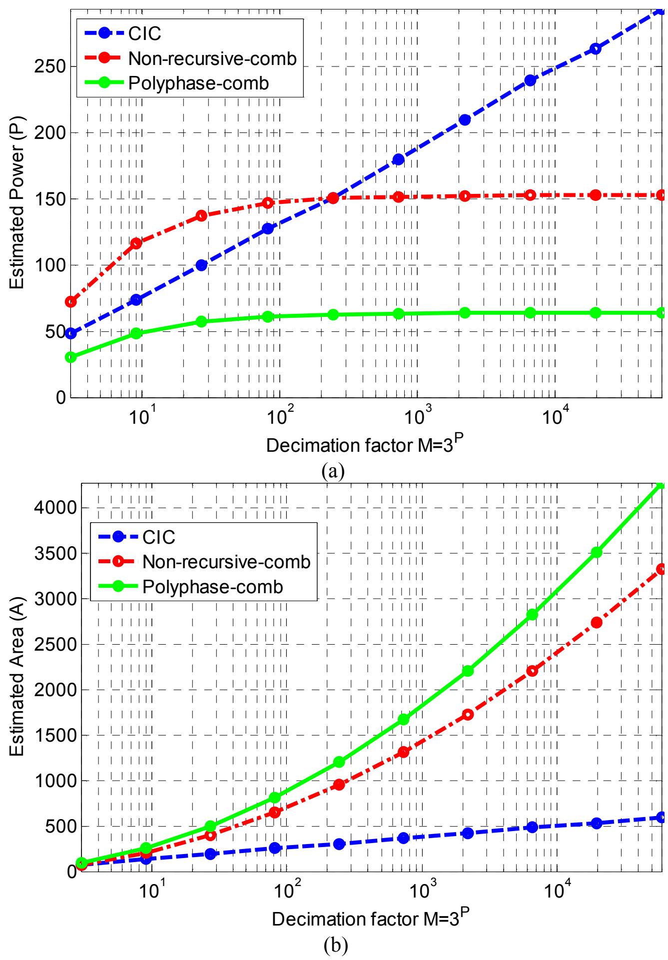


Fig. 3.Estimated (a) power and (b) area, for CIC, non-recursive-comb and polyphase-comb for M=3^P.

Similarly, Fig. 3(a) and Fig. 3(b) present the estimated power consumption and the required area, respectively, for non-recursive-comb, polyphase-comb and CIC structures considering x=3, K=3 and one bit at the input. From Fig. 3 it can be seen that for high values of the decimation factor, the estimated power of non-recursive-comb and polyphase-comb is lower than that of the CIC structure, similar to that shown for the case of x=2. Also, for high values of M the required area of polyphase-comb and non-recursive-comb is higher than that of the CIC structure. In this case, and in general for x≥3, there are some values of the decimation factor at which non-recursive-comb is less power efficient than CIC structure. This is because as x increases the number of FA and FF in the input stage of non-recursive-comb is larger than that of the required by the integrator section of the CIC structure. As a result, it is preferable to use CIC instead of non-recursive-comb for low values of the decimation factor. On the other hand, polyphase-comb structure always exhibits less power requirements than CIC for all the values of M, especially as x increases, mainly due to the frequency reduction at the first stage.

From Fig. 1 it can be concluded that the power and area estimations for x>3 has similar shape of that presented for x=3, i.e., non-recursive-comb and polyphase-comb structures are more power efficient but less area efficient than CIC for high values of M. Additionally, non-recursive-comb structure is less power efficient than CIC for low values of M.

III. Proposed Power and Area Efficient Structures

Fig. 4 shows the proposed power and area efficient decimation structures, which balances both, power and area, for large values of M. In both cases, the decimator consists of two stages. On the one hand, in first proposed structure, depicted in Fig. 4(a) and denoted as *NR-CIC-*1*,* the first stage is implemented as a non-recursive-comb structure and the second stage with the CIC structure. On the other hand, in the second proposed structure, shown in Fig. 4(b) and denoted in short as *NR-CIC*-2, the first stage is implemented as a polyphase-comb structure and the second stage with the CIC structure.

It can be demonstrated that the transfer functions of *NR-CIC*-1 and *NR-CIC*-2, referenced to the high sampling rate, can be rewritten as:

$$H(z)=\left[\left(\prod_{i=0}^{log_x(M_1)-1}\left(1\ldots+z^{-(x-1)x^i}\right)\right)\left(\frac{1-z^{-M_2}}{1-z^{-1}}\right)\right]^K, \quad (16)$$

where M_1 and M_2 are the decimation factors of the first and second stage, respectively, and K is the number of cascaded non-recursive-comb and CIC. Due to the fact that the CIC is in the second stage, in both proposed structures, its word length would require full precision,

$$B_{out}=\lceil K\cdot log_x(M_1M_2)+B_{in}\rceil. \quad (17)$$

Thus, considering (17) in (8)-(12), it can be shown that the power and area estimations for *NR-CIC*-1 are given by:

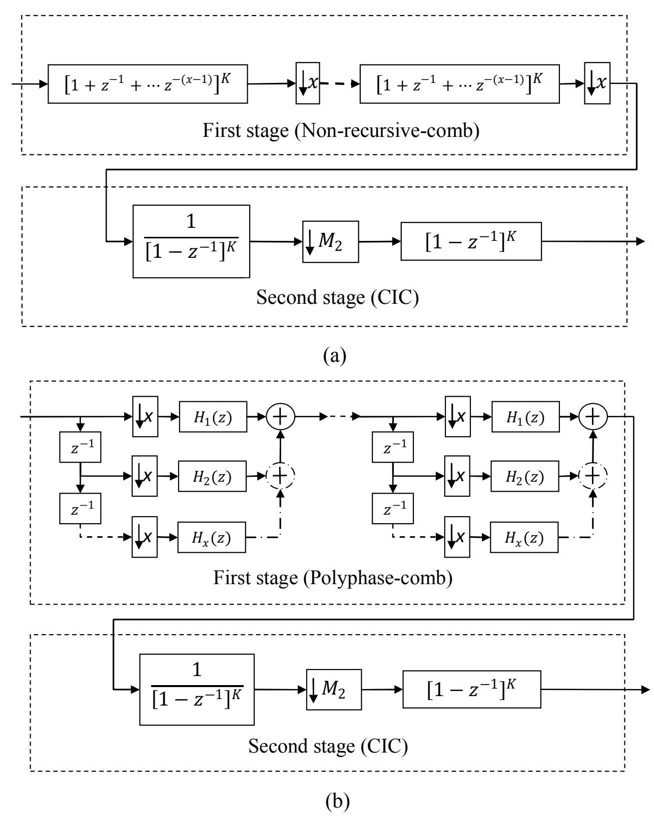


Fig. 4.Proposed decimation structures (a) *NR-CIC*-1 and (b) *NR-CIC*-2.

$$P_1 = \sum_{i=1}^{log_x(M_1)} \frac{(FA_{NCS} + FF_{NCS}) \cdot [B_{in} + K \cdot log_2(x^i)]}{x^{i-1}} + \frac{(FA_I + FF_I)B_{out}}{M_1} + \frac{(FA_C + FF_C)B_{out}}{M_1 M_2} \tag{18}$$

$$A_1 = \sum_{i=1}^{log_x(M_1)} (FA_{NCS} + FF_{NCS}) \cdot [B_{in} + K \times log_2(x^i)] + (FA_I + FF_I)B_{out} + (FA_C + FF_C)B_{out} \tag{19}$$

Similarly, using (8),(9) (14), (15) and considering (17), the power and area for the *NR-CIC*-2 can be estimated as:

$$P_2 = \sum_{i=1}^{log_x(M_1)} \frac{(FA_{PCS} + FF_{PCS}) \cdot [B_{in} + K \cdot log_2(x^i)]}{x^{i}} + \frac{(FA_I + FF_I)B_{out}}{M_1} + \frac{(FA_C + FF_C)B_{out}}{M_1 M_2} \tag{20}$$

$$A_2 = \sum_{i=1}^{log_x(M_1)} (FA_{PCS} + FF_{PCS}) \cdot [B_{in} + K \cdot log_2(x^i)] + (FA_I + FF_I)B_{out} + (FA_C + FF_C)B_{out} \tag{21}$$

Note that, in (18)-(21), the values of M_1 and M_2 are not specified. Since $M = M_1 M_2 = x^P$, it is possible to find P-1 different combinations for $M_1 M_2$, where $M_1 = x^k$ and $M_2 = x^{P-k}$, with k = 1, 2…, (P-1).

Based on the above analysis, the values of M_1 and M_2 can be found in order to get the best solution in terms of power consumption and silicon area. To this purpose, the following methodology is proposed to obtain the best value for M_1, which allows us to have an estimated power consumption as close as possible to the power consumption of either a non-recursive-comb or polyphase-comb structure, but at the same time an estimated area as close as possible to the area required for a CIC structure. In what follows, this procedure is detailed considering two different cases: x = 2 and $x \geq 3$.

A. Choice for M_1 with x=2

Let us consider the decimation factor $M=2^9=512$ and K=3, so that M_1 can take the values of 2, 4, 8, 16, 32, 64, 128 and 256. The estimated power consumption of the *NR-CIC*-1 for those values of M_1 is plotted in Fig. 5(a). The reference values of power consumption for a purely non-recursive-comb and CIC structures, both with M=512 (see Fig. 2(a)), are also shown. From Fig. 5(a), it can be seen that for $M_1 \geq 4$ the *NR-CIC*-1 has the same power consumption as that of non-recursive-comb structure. In fact, the power consumption in the *NR-CIC*-1 is slightly lower than that in non-recursive-comb when $8 \leq M_1 \leq 32$.

TABLE I

ADDERS AND FLIP-FLOPS REQUIRED IN POLYPHASE-COMB FOR X=3.

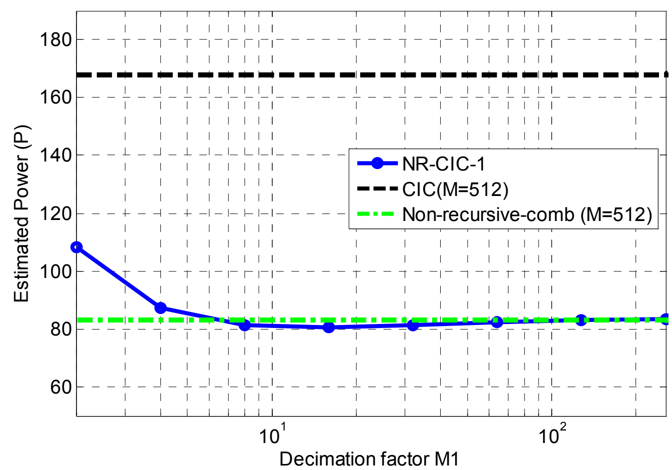
K	FA on structure	FA on coefficients	FA_{PCS}	FF_{PCS}
2	4	1	5	2
3	6	5	11	4
4	8	4	12	6
5	10	21	31	8

Similarly, the used area of the *NR-CIC*-1 as a function of the first decimation factor M_1 is shown in Fig. 5(b). The referent values of area for non-recursive-comb and CIC structures, both for M=512 (see Fig. 2(b)), are also shown. From Fig. 5(b) it can be seen that in order to obtain a low area in the *NR-CIC*-1, similar to that in the CIC structure, low values of M_1 have to be used.

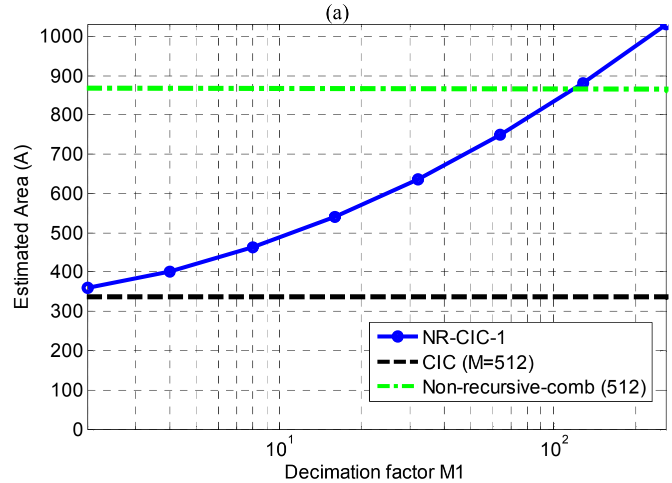
Observing Figs. 5(a) and 5(b), it can be concluded that a good choice for M_1 is 4, and therefore M_2=128. In this way, the *NR-CIC*-1 exhibits the low power characteristic as a non-recursive-comb-structure and a low area as the CIC structure. Thus, following a similar procedure, the best choice of M_1 can be found for higher decimation factors M. For example, the chosen values of M_1 are 4 and 8 for M=512, 1024 and M=2048, 4096, respectively.

A similar procedure can be followed for the choice of the best value of M_1 in the *NR-CIC*-2. Considering that M=512, Figs. 6(a) and 6(b) show the influence of M_1 in the power and area requirements of *NR-CIC*-2, respectively. For the sake of completeness, Fig. 6 depicts also the reference values of power and area for polyphase-comb and CIC structures with M=512.

By observing Fig. 6 and following the proposed



(a)



(b)

Fig. 5.Estimated (a) power and (b) area, for *NR-CIC*-1, $M=2^9$.

methodology for choosing the value for M_1, it can be concluded that for M=512 the best value of M_1 is 8 in the case of the *NR-CIC*-2 topology. In this way, the *NR-CIC*-2 exhibits the low power characteristic as a polyphase-comb structure and a low area as the CIC structure. It can be demonstrated, that the best choices for M_1 are 8 and 16 for M=512, 1024 and M=2048, 4096, respectively.

B. Choice of M_1 with $x \geq 3$

Let us consider first a decimation factor that is a power of 3, taking as an example $M=3^{10}$=59049. In this case, M_1 can take the values 3, 9, 27... 3^9. The reason for choosing those values of M_1 lies on the fact that for decimation factors lower than 3^8=6561, the required power of non-recursive-comb is larger than in the equivalent CIC structure, (see Fig. 3(a)). If the power estimation of non-recursive-comb is larger than that of the CIC, then the *NR-CIC*-1 will not be power efficient since its first stage is based on non-recursive-comb structure.

Fig. 7(a)-(b) show the estimated power and area of the *NR-CIC*-1 structure, considering different possible values of M_1, along with the reference values for non-recursive-comb and CIC structures with $M=3^{10}$. Observing Fig. 7 it can be concluded that the best choice for M_1 is 3, and hence $M_2=3^9$=19683. In this way, the *NR-CIC*-1 exhibits the same low power characteristic as a non-recursive comb structure and at the same time low area as the CIC structure. However, although, in this example, the *NR-CIC*-1 is both power- and area-efficient, a decimation factor larger than 3^{10} is unfeasible for practical applications in ΣΔ-ADCs, where such oversampling ratios would result in excessively power consumption in the sigma-delta modulator.

On the other hand, since polyphase-comb requires less power than CIC for all values of M (see Fig. 2(a) and Fig. 3(a)) the *NR-CIC*-2 topology can be both power- and area-efficient for lower values of the decimation factor, or equivalently lower values of P.

Let us consider K=3, $M=3^6$=729, which is a feasible value for ΣΔ-ADCs in low-frequency applications. Fig. 8(a) and 8(b) show the estimated power and area, respectively, for the *NR-CIC*-2 topology, as a function of the first decimation factor. The reference values for non-recursive-comb and CIC structures with $M=3^6$ are also presented.

Following the proposed methodology, the best value for M_1 is 9, and therefore M_2=81. With those values, the *NR-CIC*-2 topology shows a similar power efficiency as that of polyphase-comb structure, and an area efficiency similar to CIC structure. Therefore, the proposed *NR-CIC*-2 structure results in a more efficient solution in terms of power and active area, whenever the values of the decimation factors are lower than that required for the *NR-CIC*-1 topology.

From (8)-(15), (18)-(21) it can be concluded that the power and area estimations in *NR-CIC*-1 and *NR-CIC*-2 for x>3 has similar behavior to that presented for x=3. It means that, although the *NR-CIC*-1 is power efficient for large values of

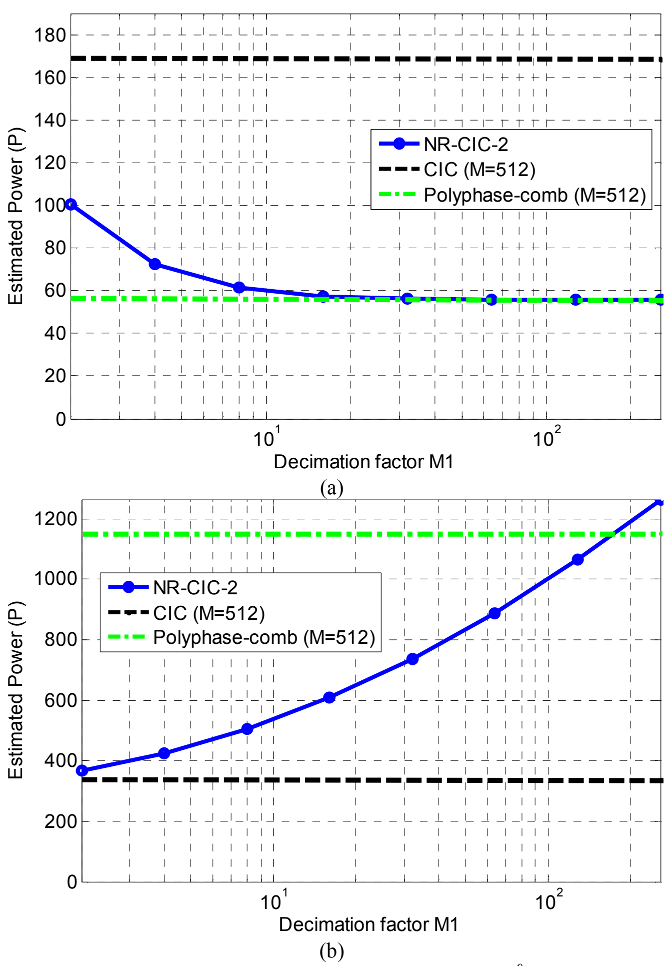


Fig. 6.Estimated (a) power and (b) area, for *NR-CIC*-2, $M=2^9$.

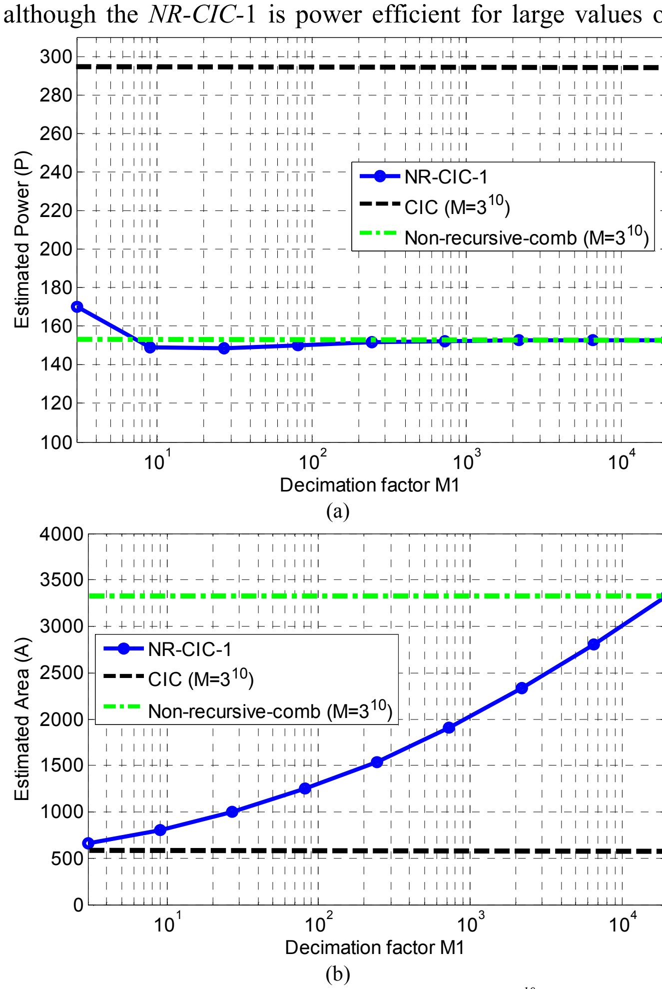
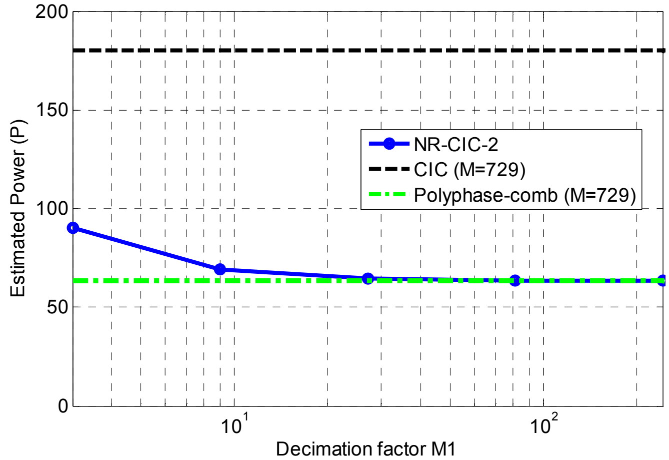
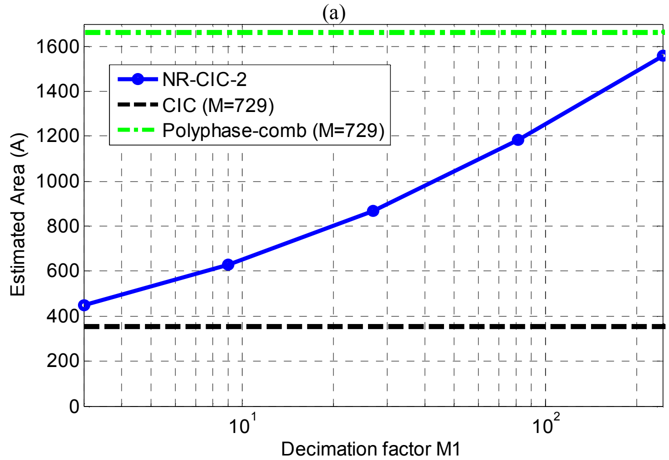


Fig. 7.Estimated (a) power and (b) area, for *NR-CIC*-1, $M=3^{10}$.



(a)



(b)

Fig. 8.Estimated (a) power and (b) area, for *NR-CIC*-2, M=3^6.

M, in the case x≥3, the decimation factors becomes of unpractical application in ΣΔ-ADCs. Therefore, for practical implementations of ΣΔ-ADCs, the *NR-CIC*-1 topology is limited to be used with x=2. On the other hand, the *NR-CIC*-2 structure is both power- and area-efficient for x≥3, even at low values of P, thanks to the fact that polyphase-comb power is always less than that of the equivalent CIC, (see (8)-(15) and Fig. 1). Note that, although *NR-CIC*-1/*NR-CIC*-2 can be used to obtain both power- and area-efficient implementations, if we consider x≥11, the resulting decimation factors would be relatively high for typical ΣΔ-ADC applications.

Moreover, even though *NR-CIC*-1 and *NR-CIC*-2 are power and area efficient, their magnitude responses exhibit a low attenuation in the folding bands as the non-recursive-comb and CIC structures. Next section introduces a slight modification in *NR-CIC*-1 and *NR-CIC*-2 in order to improve the alias rejection in the first folding band.

IV. Proposed Structures with Improved Alias Rejection

The magnitude response of both *NR-CIC*-1 and *NR-CIC*-2 is equal to that of the comb filter:

$$\left|H(e^{j\omega})\right| = \left|\frac{1}{M}\frac{\sin\left(\frac{M\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}\right|^{K}. \tag{22}$$

Thus, the first folding band of the proposed structures provides the Worst Case Aliasing (WCA) attenuation, which occurs at the following angular frequency [15]:

$$\omega_1 = \frac{2\pi}{M} - \frac{\pi}{RM}, \tag{23}$$

where R stands for the residual decimation factor used by the decimation filter that follows the proposed structures.

In order to improve the WCA attenuation in the proposed structures, a basic non-recursive-comb filter given by

$$G(z) = \frac{1}{x}\left(1 + z^{-1} + z^{-2} + z^{-3} \ldots + z^{-(x-1)}\right) \tag{24}$$

will be considered.

The expression of $G(z)$ expanded by an integer N is given by:

$$G(z^N) = \frac{1}{x}\left(1 + z^{-N} + z^{-2N} + z^{-3N} \ldots + z^{-(x-1)N}\right), \tag{25}$$

which has the following frequency response:

$$\left|G(e^{j\omega N})\right| = \left|\frac{1}{x}\frac{\sin\left(\frac{xN\omega}{2}\right)}{\sin\left(\frac{N\omega}{2}\right)}\right|. \tag{26}$$

It can be noted from (22) and (26) that the zeros of the magnitude response of *NR-CIC*-1 and *NR-CIC*-2 are at integer multiples of 2π/M, while the zeros in the expanded filter are at integer multiples of 2π/xN. Therefore, by cascading (25) with (15), an improved version of the *NR-CIC*-1 structure – shown in Fig. 9(a) and denoted in short as *NR-CIC*-3 – can be synthesized. Note that this improved structure has K_2 extra zeros in the first folding band provided that N=M/x. As a result, an increase of the attenuation in the most critical band is achieved. Proceeding in a similar way, an improved version of *NR-CIC*-2 – denoted in short as *NR-CIC*-4 and shown in Fig. 9(b) – is obtained, which also present K_2 extra zeros. Therefore, in order to improve alias rejections of both *NR-CIC*-3 and *NR-CIC*-4 structures, suitable values of K_2 need to be derived for a given WCA attenuation factor, as discussed in the next section.

V. Proposed Design Procedure

The frequency response of the proposed *NR-CIC*-3 and *NR-CIC*-4 structures can be obtained by combining (22) and (26):

$$\left|H_{3,4}(e^{j\omega})\right| = \left|\frac{1}{M}\frac{\sin\left(\frac{M\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}\right|^{K_1}\left|\frac{1}{x}\frac{\sin\left(\frac{xN\omega}{2}\right)}{\sin\left(\frac{N\omega}{2}\right)}\right|^{K_2} \tag{27}$$

where K_1 and K_2 are the number of cascaded filters in the *NR-CIC*-1/*NR-CIC*-2 and expanded-comb, respectively.

By evaluating (27) at the frequency given in (23) the magnitude response of *NR-CIC*-3 and *NR-CIC*-4 is given by:

$$\left|H_{3,4}(e^{j\omega_1})\right| = \left[\frac{1}{M}\frac{\sin\left(\frac{\pi(2R-1)}{2R}\right)}{\sin\left(\frac{\pi(2R-1)}{2MR}\right)}\right]^{K_1}\left[\frac{1}{x}\frac{\sin\left(\frac{\pi(2R-1)}{2R}\right)}{\sin\left(\frac{\pi(2R-1)}{2xR}\right)}\right]^{K_2} \tag{28}$$

Assuming that M>>R, the above expression simplifies into:

$$\left|H_{3,4}(e^{j\omega_1})\right| \approx \left[\frac{\sin\left(\frac{\pi(2R-1)}{2R}\right)}{\left(\frac{\pi(2R-1)}{2R}\right)}\right]^{K_1}\left[\frac{1}{x}\frac{\sin\left(\frac{\pi(2R-1)}{2R}\right)}{\sin\left(\frac{\pi(2R-1)}{2xR}\right)}\right]^{K_2}, \quad (29)$$

which depends on K_1, K_2, x and R, but it does not depend on M.

If $K_1=K$ is assumed, it can be noticed from (22), (26) and (29) that each added K_2 in the *NR-CIC*-3/*NR-CIC*-4 will improve the WCA, with respect to *NR-CIC*-1/*NR-CIC*-2, by an amount that depends only on the value of x and R. Table II illustrates the WCA improvement (WCAI) for different values of x and R.

Denoting the desired WCA in dB as A_D, the number of K_2 filters can be determined as:

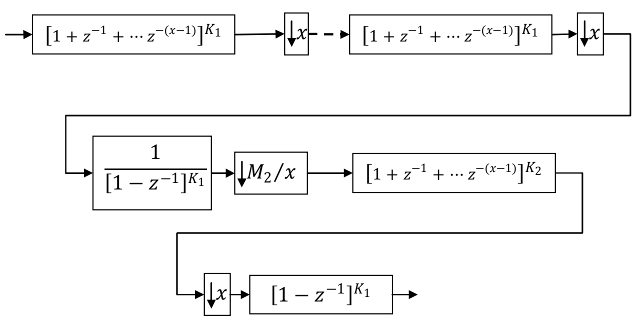
$$K_2 = \left\lceil \frac{A_D - A_{NR-CIC-1,2}}{WCAI} \right\rceil, \quad (30)$$

where $\lceil . \rceil$ is the ceiling function; $A_{NR\text{-}CIC\text{-}1,2}$ is the WCA in the *NR-CIC*-1 or *NR-CIC*-2, and WCAI is the improvement of each K_2 filter, which can be obtained from Table II or from (29).The maximum number of K_2 filters that can be added will be limited by the obtained attenuation in the *x-th* folding band, which, considering $M>>R$, is given by:

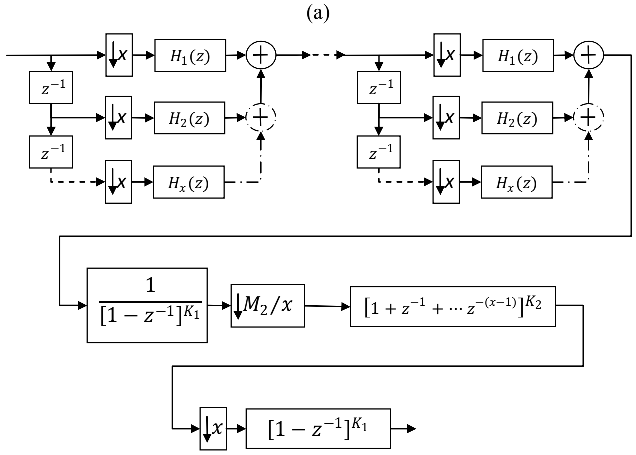
$$\left|H_{3,4}(e^{j\omega_x})\right| \approx \left[\frac{\sin\left(\frac{\pi(2xR-1)}{2R}\right)}{\left(\frac{\pi(2xR-1)}{2R}\right)}\right]^{K} \cdot \left[\frac{1}{x}\frac{\sin\left(\frac{\pi(2xR-1)}{2R}\right)}{\sin\left(\frac{\pi(2xR-1)}{2xR}\right)}\right]^{K_2}. \quad (31)$$

In order to ensure that at least the same WCA occurs in all the folding bands of the *NR-CIC*-3/*NR-CIC*-4 the following relationship must be satisfied:

$$\left|H_{3,4}(e^{j\omega_x})\right| \leq \left|H_{3,4}(e^{j\omega_1})\right|. \quad (32)$$



(a)



(b)

Fig. 9.Improved versions of proposed structures (a) *NR-CIC*-3 and (b) *NR-CIC*-4.

After some transformations in the above expression, the following condition can be found:

$$\frac{K_1}{K_2} \leq 20log\left\{\frac{\left[\frac{1}{x}\frac{\sin\left(\frac{\pi(2xR-1)}{2R}\right)}{\sin\left(\frac{\pi(2xR-1)}{2xR}\right)}\right]}{\left[\frac{1}{x}\frac{\sin\left(\frac{\pi(2R-1)}{2R}\right)}{\sin\left(\frac{\pi(2R-1)}{2xR}\right)}\right]} - \frac{\left[\frac{\sin\left(\frac{\pi(2R-1)}{2R}\right)}{\left(\frac{\pi(2R-1)}{2R}\right)}\right]}{\left[\frac{\sin\left(\frac{\pi(2xR-1)}{2R}\right)}{\left(\frac{\pi(2xR-1)}{2R}\right)}\right]}\right\}. \quad (33)$$

Fig. 10 depicts the condition in (33) as a function of x and R. In general it can be seen that for a given value of R and as x increases, the cascaded factor K_2 can be larger, i.e. it can provide more WCA improvement without being worried about the attenuation in the subsequent folding bands. On the other hand, for a given value of x and as R raises the number of K_2 filters that can be added is reduced, i.e. the possible WCA improvement will be reduced. Note that the magnitude responses of both improved structures (*NR-CIC*-3 and *NR-CIC*-4) are equal to the magnitude response of a two-stage comb structure in which the first stage has the decimation factor of M/x and the second stage has a decimation of x and an increased cascade. As a result, not only first, but also all folding bands will be improved except those which are multiple of x [16].

Example 1: Let us consider the *NR-CIC*-1 structure of Fig. 4(a) with M_1=4, M_2=128, K=3, and R=2, which has the WCA of -30dB. However, a WCA of at least -45dB is required. To this end, we use the *NR-CIC*-3 structure of Fig 9(a) and with (30) we calculate the required K_2 filters as:

$$K_2 = \left\lceil \frac{30-45}{8.34} \right\rceil = \lceil 1.79 \rceil = 2$$

TABLE II
WCA IMPROVEMENT AS A FUNCTION OF X AND R FOR EACH ADDED K_2.

x	WCAI (dB)			
	R=2	*R*=4	*R*=8	*R*=16
2	-8.32	-14.19	-20.17	-26.18
3	-9.54	-15.87	-22.13	-28.29
5	-10.13	-16.68	-23.06	-29.98
7	-10.28	-16.90	-23.31	-29.56
11	-10.38	-17.03	-23.47	-29.73

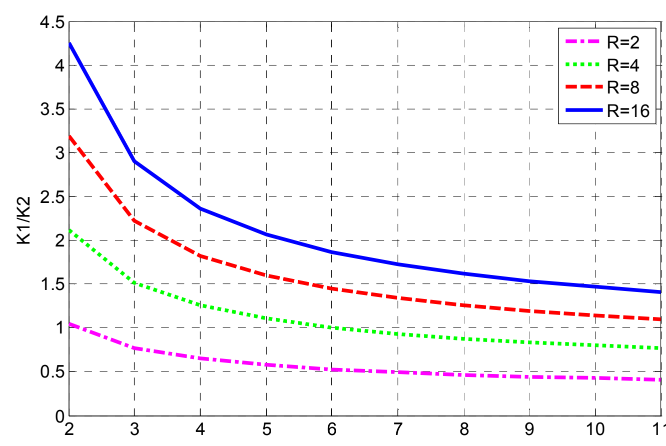


Fig. 10.Graphical representation of condition in (33).

The overall magnitude responses of *NR-CIC*-1 and *NR-CIC*-3 along with a zoom in the first folding band are shown in Fig. 11(a). Fig. 11(b) shows the zoom in the first eight folding bands, where it can be seen that all odd folding bands are improved.

A. Power reduction in NR-CIC-3 and NR-CIC-4

The main contribution to the power consumption in the proposed *NR-CIC*-1 and *NR-CIC*-2 structures comes from the first stage, non-recursive-comb and polyphase-comb, respectively, which work at the highest sampling frequency. In Section II it was demonstrated that reducing the input frequency before any filtering, provides a power requirements reduction, as in polyphase-comb. Another way to reduce the power requirements is by reducing the number of cascaded stages K_1 in *NR-CIC*-3 and *NR-CIC*-4, i.e. reducing the number of additions performed at the input stages. However, such a reduction of K_1 produces a reduction in the WCA attenuation. Nevertheless, the added K_2 filters in *NR-CIC*-3 and *NR-CIC*-4 provides an additional degree of freedom that can be used to compensate the reduction of K_1 and, hence, reduce the power requirements while retaining a minimal attenuation in all folding bands.

From (22) and (29) it can be seen that for a given value of x and R, the WCA in *NR-CIC*-1/*NR-CIC*-2 depends only on K, but in *NR-CIC*-3/*NR-CIC*-4 depends on K_1+K_2. Therefore, it is possible to obtain the same WCA in *NR-CIC*-3/*NR-CIC*-4 as in *NR-CIC*-1/ *NR-CIC*-2 by using different combinations for K_1 and K_2. However, only the combinations that satisfies the condition (33) can be used.

From Fig. 10 it can be seen that for a given value of R, as x raises the ratio K_1/K_2 decreases. This means that we can reduce the value of K_1 and compensate this by K_2. Actually, whenever $K_1/K_2<1$ the number of K_1 cascades can be lower than K_2. The latter would lead to a good power savings while keeping the WCA in all folding bands.

On the other hand, for a given value of x and as R increases, the ratio K_1/K_2 increases, and consequently K_1 can be reduced and compensated by an increase in K_2 but only for $K>10$, which is not of a practical case when narrow bands are used. Therefore, the low power structure has a practical limitation for residual decimation factors R≤4.

Example 2: Let us consider the *NR-CIC*-2 with $M_1=5$, $M_2=625$, $R=2$ and $K=6$, with those values the WCA is around −60dB. However, we are interested in reducing the power consumption while keeping at least the WCA of −60dB in all folding bands. To this end, we use the *NR-CIC*-4 structure, where the same WCA can be obtained with $K_1=3$ and $K_2=3$, and at the same time those cascades satisfy (33). Fig. 12(a) and Fig. 12(b) present the magnitude response of *NR-CIC*-2 and *NR-CIC*-4 for the first ten folding bands and a zoom in the first and fifth folding band, respectively. From Fig. 12 it can be seen that the minimum attenuation is maintained in all folding bands and using (8)-(15), (18)-(21) it can be found that the reduction in K_1 from 6 to 3 leads to a power savings of around 45%.

VI. IMPLEMENTATION

In order to prove the power and area efficiency of the proposed structures we have taken as an example the *NR-CIC*-1 structure for M=512, K=3 and *NR-CIC-3* with M=512, K_1=3, K_2=2 and implemented it in VHDL, at Register Transfer Level (RTL). The VHDL models of the filters, including the frequency divider, have been synthesized into standard cells of

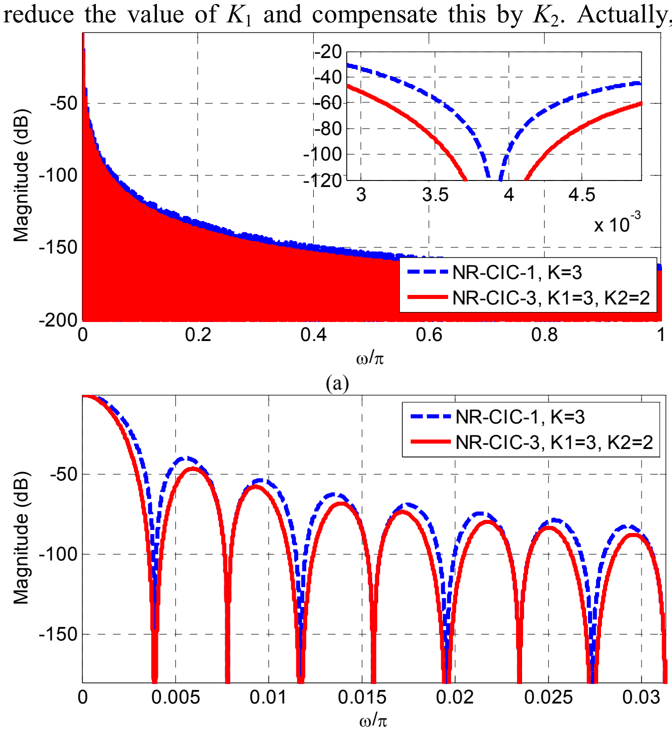


Fig. 11. (a) Overall magnitude response for *NR-CIC*-1 and *NR-CIC*-3 with M=512 and (b) zoom in the first eight folding bands.

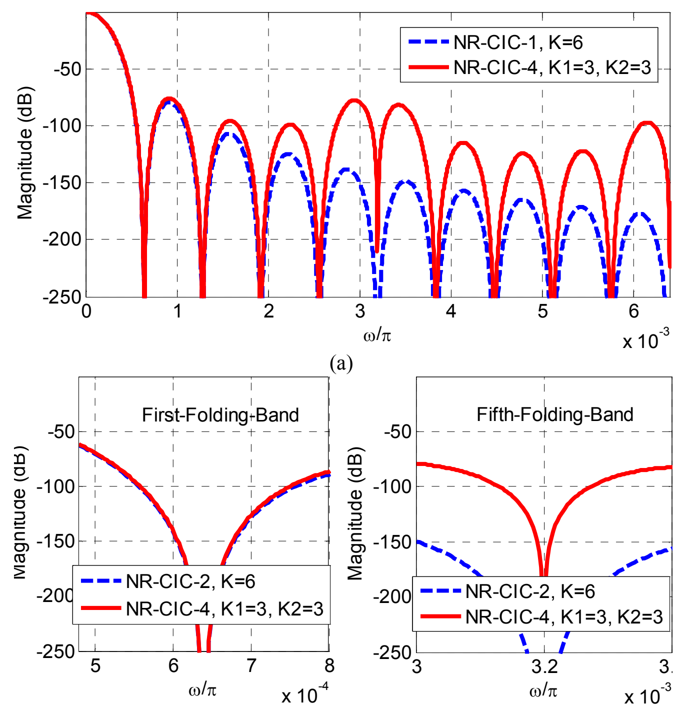


Fig. 12. (a) Magnitude response of the first ten folding bands of *NR-CIC*-2 and *NR-CIC*-4 with $M=5^5$ and (b) zoom in the first and fifth folding band.

a 0.18μm CMOS technology. The obtained transistor-level models of filters, without parasitic effects, were used in Synopsys HSPICE in order to simulate the power consumption with a power-supply of 1.8V. The input signal used to verify the performance of the filters was the output bitstream of an ideal, first-order one-bit ΣΔ modulator, in which the input is a sine wave of 9.76 kHz and the modulated output has a sampling frequency of 10MHz, and an OSR=512. The obtained layouts were used to measure the used area of each filter.

Table III presents a summary of power consumption and used area for the non-recursive-comb, CIC and *NR-CIC*-1 for M=512 and K=3. It can be seen that the CIC filter requires more power than the others, but it uses less area. The non-recursive-comb has less power consumption than the CIC, but uses more area. The *NR-CIC*-1 has similar power consumption than the non-recursive-comb and almost the same used area of the CIC filter as was predicted in previous sections.Finally, Table IV presents a summary of power consumption and the used area for *NR-CIC*-1 and *NR-CIC*-3, where it can be seen that *NR-CIC*-3 has an increase in power consumption of 1% compared with *NR-CIC*-1. Additionally, *NR-CIC*-3 has a relative increase of 20% in the used area.

TABLE III
SUMMARY OF AREA AND POWER CONSUMPTION OF NON-RECURSIVE-COMB, CIC AND *NR-CIC*-1.

Structure with M=512, K=3.	Total Power (μW)	Total Area (μm^2)
Non-recursive-comb	226	424,569
CIC	408	326,041
NR-CIC-1 M_1=4, M_2=128	235	339,309

TABLE IV
SUMMARY OF AREA AND POWER CONSUMPTION OF *NR-CIC*-1 AND *NR-CIC*-3.

Proposed structures with M_1=4, M_2=128	WCA (dB)	Total Power (μW)	Extra Power (%)	Total Area (μm^2)	Extra Area (%)
NR-CIC-1 K=3	-30	235	0	339,309	0
NR-CIC-3 K_1=3, K_2 =2	-46	238	1	423,832	20

VII. CONCLUSION

This paper presented several novel two-stage efficient comb-based decimation structures for high values of the decimation factors, which can be presented as power of any prime number x. One of the proposed structures, denoted in this work as *NR-CIC*-1 structure, takes the power benefits of non-recursive-comb and the area efficiency of CIC structure. The detailed power and area analysis of the proposed structure in term of the values of x, is included. On the base of this analysis, the choice of the decimation factors for both stages, which makes the best balance of area and power efficiency, is elaborated. Another novel two-stage power and area efficient structure, which takes the benefit of the polyphase decomposition of the non-recursive part, is also proposed (structure *NR-CIC*-2). The detailed analysis of the area and power consumption in terms of x is also given in order to provide the necessary tools to synthesize such structures. The presented modified structures (*NR-CIC*-3 and *NR-CIC*-4) have improved alias rejections in the first folding band, and in all other folding bands which are not factors of x. The compensation of the passband droop has not been considered, because the problem can be solved by cascading any simple known *multiplierless* compensator at lower rate. The efficiency of the proposed structures has been validated in VHDL, considering a 0.18μm CMOS technology, taking x=2 and M=512 as an example. The obtained results confirm that the proposed structures are efficient solutions for the implementation of ΣΔ ADCs converters with high values of the oversampling ratio.

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