NARROWBAND NOISE INTERPOLATED AND TRANSLATED BY MULTIRATE PROCESSING TECHNIQUES

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ABSTRACT

A common signal processing task is to generate a narrow noise time series with specified passband and stopband bandwidths sampled at a specified sample rate. At first glance, this appears to be a straightforward problem. We simply generate samples of the broadband noise at the desired sample rate and then band limit the noise with a digital filter deigned to match the passband and stopband specifications. We show here that this approach is quite inefficient and in fact is untenable when there is a large ratio of sample rate to bandwidth. We then show a most efficient technique for which baseband noise is generated at a low sample rate and is then interpolated up to the desired sample rate and desired center frequency.

Keywords: Narrowband Noise, Interpolator, Band Centered

1. INTRODUCTION

There are many simulation and signal generation applications that require a narrow noise time series of a specified bandwidth centered at a selected center frequency and sampled at a specified sample rate. The most obvious option, and as it happens the most inefficient option, is to generate a wide bandwidth noise series at the desired sample rate then limit the bandwidth with a baseband low pass filter and then finally use a complex heterodyne to translate the baseband centered spectrum to the desired center frequency. This option is shown in figure 1a. To illustrate a specific example we will form a 1-MHz bandwidth noise signal sampled at 800 MHz and centered at an arbitrary frequency, say 170.5 MHz. A sketch of the spectra at successive positions in the processing stream is shown in figure 1b.

The problem with this solution is it cannot be implemented. The baseband low pass filters are not implementable. Let's consider a specific filter specification and then examine the filter options. In particular, we want a filter with 3 dB pass-band edge of 0.5 kHz, stop-band edge of 0.61 kHz and a stopband attenuation of 90 dB. A FIR filter meeting these specifications operating at 800 MHz sample rate would require approximately 25,000 taps. It is not feasible to operate a pair of 25,000 tap filters at an 800 MHz sample rate.

If we examine the IIR option we find that a 13-order Elliptic filter can meet the specifications. There are two implementation problems with the IIR filter. The first is the coefficient bit width required to place the filter poles in a very small region near DC; 24 bits are required to position these roots. The second is the numerical growth of the filter internal state due to the extremely small distance between its poles and the unit circle as shown in figure 2. The pole cluster crowded into the small region near DC is the consequence of the large ratio of sample rate to filter bandwidth.

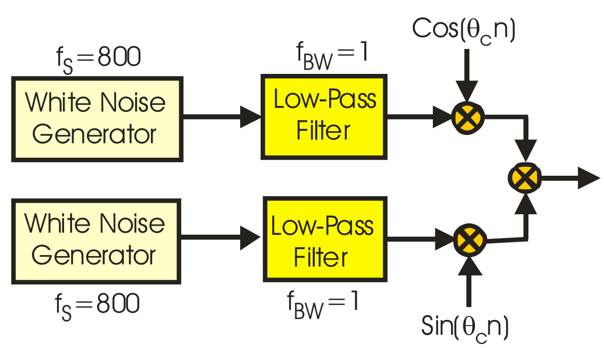

Figure 1a. Forming Band Centered Narrow Band Noise with Pair of Low Pass Filters and I-Q Heterodyne

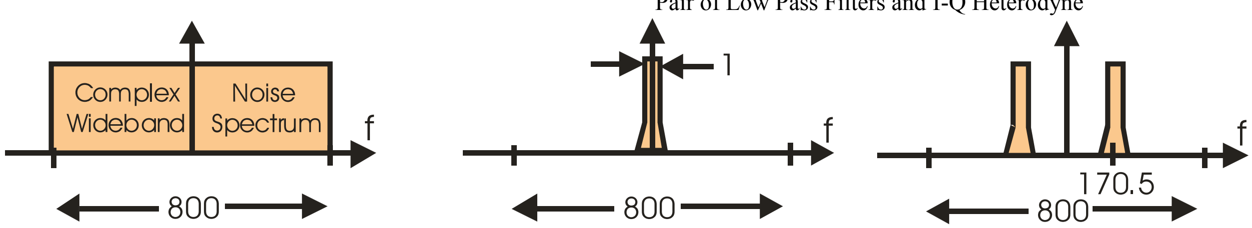

Figure 1b. Spectra of Broadband Input Noise, Baseband Filtered Noise, and I-Q Up-Converted Band Centered Noise

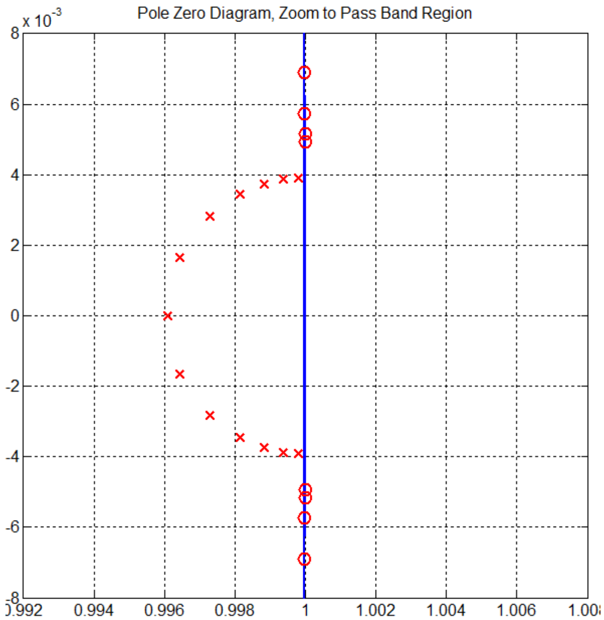

Figure 2. Pole Zero Diagram of 13-Order Elliptic Low-Pass Filter with Sample Rate 800 Times Bandwidth.

Internal states of this filter exhibit 6-orders, or 20 bits, of growth relative to input samples. For 12 bit input samples, the internal states require 32 bit registers. Thus the IIR filter would have to form 13 poles and 13 zeros, nominally with a multiply and add per root, using extended precision 24 by 32 bit multipliers and 32 bit adders at an 800 MHz sample rate. The extended precision arithmetic triples or quadruples the arithmetic workload per output sample which is already stressed by the high output sample rate. This proves to be a very expensive option and we seek an option with reduced computational costs.

2. SECOND DESIGN APPROACH

We now consider an alternate approach to the task of forming the narrow band noise at a high sample rate. In this option we form the narrow band base-band noise series at a low sample rate, interpolate up to the required sample rate and then heterodyne the up-sampled baseband sequence to the desired center frequency. This option is shown in figure 3a. A sketch of the spectra at successive positions in the processing stream is shown in figure 3b. A FIR filter length to perform the 1-MHz spectral shaping with 110 kHz transition bandwidth is on the order of 100 taps while a two-path 13-th order IIR filter can perform the same filtering with 13 multiplies per output sample. The filter length per path for an 80-dB dynamic range 200-path polyphase interpolator up sampling filter from an initial sample rate of 4 to an output sample rate of 800 is approximately 12 taps.

When we amortize the 13-th order IIR low pass filter or the 100 tap FIR low pass filter workload over the 200 output samples from the interpolator and include the output heterodyne, we find the total workload per output sample is less than 14 multiplies per output sample. This is an 1800-to-1 reduction in workload compared to the bandwidth reducing filter operating at the high output sample rate as opposed to a low input sample rate.

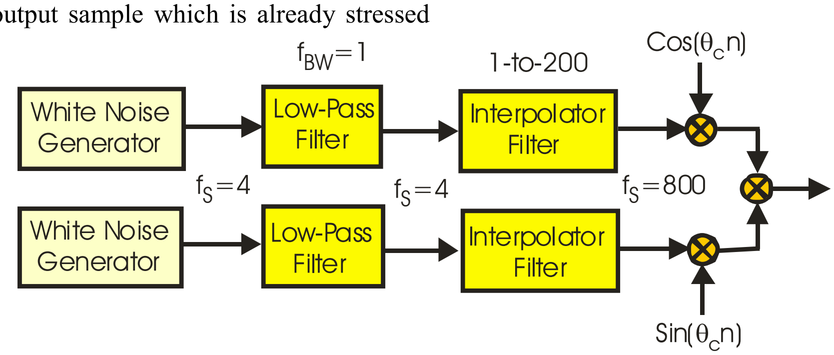

Figure 3a. Forming Band Centered Narrow Band Noise with Low-Pass Filter, Interpolating Filter, and Heterodyne

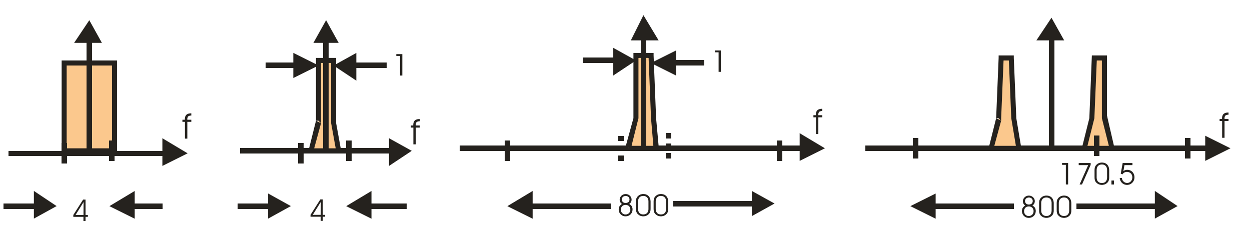

Figure 3b. Spectra at Successive Points in Processing Stream

3. THIRD DESIGN APROACH

A third option modifies the previous technique by embedding the heterodyne operation in the interpolator by replacing the weights of the low-pass prototype filter with the weights of a band-pass prototype filter. When the center frequency of the band shifted filter is a multiple of the input sample rate, the heterodyne factors out of the polyphase arms as a complex rotator per path and the filter operates as a single channel synthesis channelizer. In this form, the channelizer performs the frequency shift as an alias operation to the channel center frequency during the up-sampling process. The alias based heterodyne can extract a copy of the input spectrum from any multiple of the input sample rate. This is the dual of the down sample operation in which every multiple of the output sample rate aliases to baseband when down sampled.

The up-conversion process obtained by extracting the spectrum from the selected Nyquist zone during the up-sampling process requires a minor phase rotation modification to enable translation to any arbitrary center frequency which differs from a multiple of the input sample rate. There are a number of methods to insert this phase rotation in the signal flow path to obtain the desired translation to an arbitrary center frequency. The one we present here is a pre-process frequency offset applied at baseband prior to the up-sampling channelizer. We can think of the preprocessor as the fine resolution component of the frequency shift and the alias offset as the course resolution component. The block diagram of this version of the up-sample and translate process is shown in figure 4.

In this version the interpolation is performed in three stages. The first stage performs a 4-channel analysis and 4-to-2 down sampling of the input baseband noise signal that has been frequency shifted by up to ± 1 MHz by the input heterodyne. The analysis filter delivers three adjacent frequency bins centered at -1, 0, and 1 MHz containing the arbitrarily shifted baseband signal to the synthesis channelizer. Figure 5a shows the spectrum of the baseband narrow-band noise and the spectra of the three adjacent output channel filters -1, 0, and +1 of the 4-path analysis filter. The filter bank performs a 4-to-2 down sample with Nyquist -6 dB bandwidths of 1 MHz so that each port has a nominal 1 MHz two sided bandwidth sampled at 2 MHz. Figure 5b shows the spectrum of the narrowband input noise following the 0.5 MHz heterodyned frequency offset. We also see the spectral content of the three output ports of the 4-path analysis filter. The translated input signal overlaps the frequency bins 0 and +1 and parts of its spectrum now reside in each of their outputs. The content of these adjacent bins will be reassembled in the next stage perfect reconstruction 80-path synthesis filter.

The second stage filter is an 80-path channelizer that performs a 1-to-40 up-sampling and course frequency offset to any of the 80 offset channels centered at integer multiples of 2-MHz input sample rate, say to 10 MHz. The input spectrum, original offset from 0 by +0.5 MHz now resides with a +0.5 MHz offset from 10 MHz at 10.5 MHz. We will see why the 10.5 MHz when we examine the next 1-to-10 interpolator.

The third and final stage interpolator is a 10-path band-pass interpolator (as opposed to low-pass). As seen in figure 6, this prototype filter has the appearance of sinc function with equally spaced spectral zeros separated by 2π/10. The weight vector of the prototype filter is heterodyned to the center frequency of the 1-to-10 up sampled signal, easily visualized as containing 10 replicate spectra obtained by 1-to-10 zero packing of the input sequence. Of course we do not zero pack the signal; rather we polyphase partition the heterodyned filter. The filter spectrum so heterodyned places

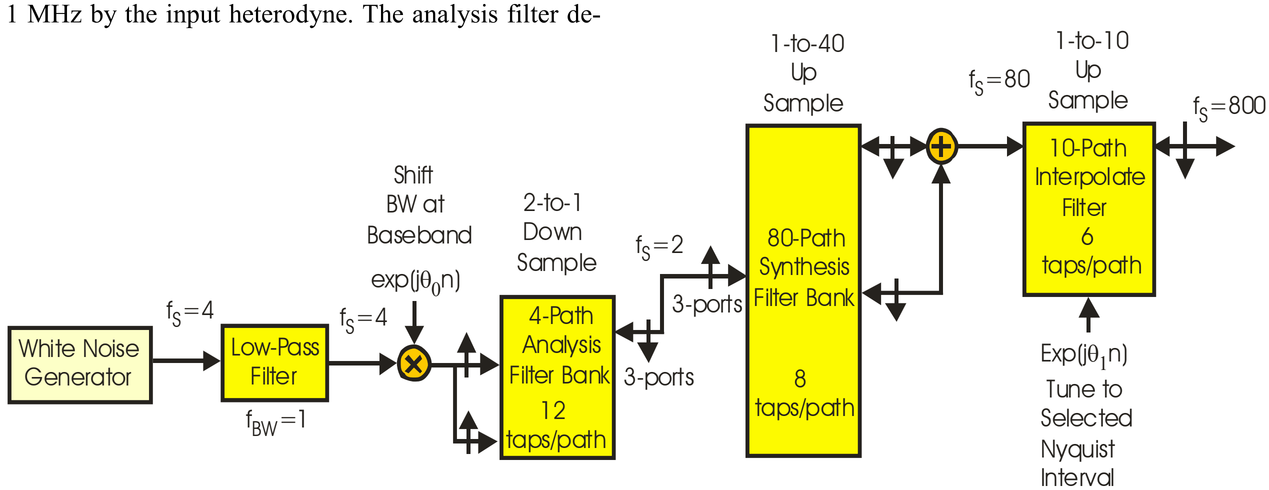

Figure 4. Forming Band Centered Narrow Band Noise with Low-Pass Filter, Fine Resolution Baseband Heterodyne, 2-to-1 Down Sample 4-Path Analysis Filter, 1-to-40 Up Sample 80-Path Channelizer Filter, and Final 1-to-10 Band Pass Interpolating Filter

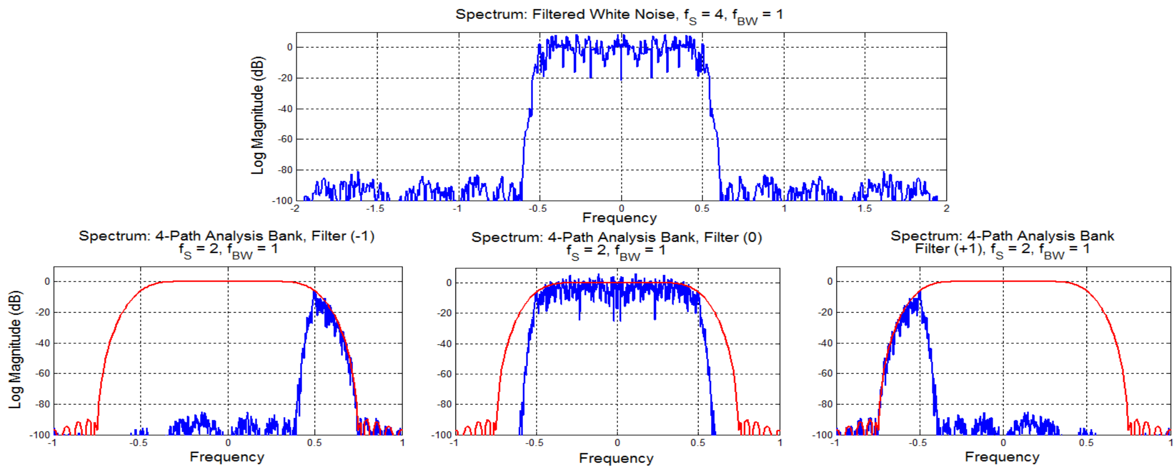

Figure 5a. Input Spectrum, Baseband Narrow Band Noise, f_S=4, f_{BW}=1, f_C=0 and Three Channels -1,0,+1 of 4-Path, 2-to-1 Down Sample Analysis Filter Bank

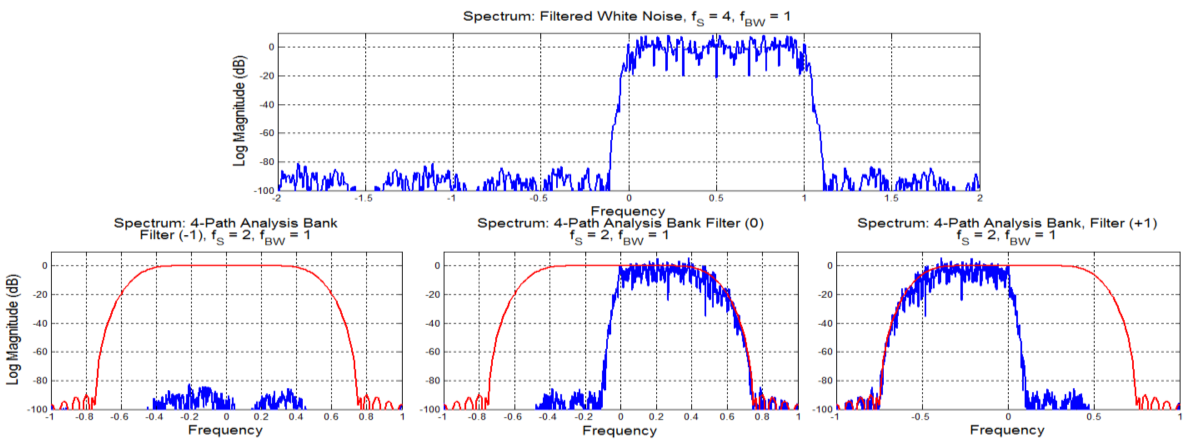

Figure 5b. Input Spectrum, Baseband Narrow Band Noise, f_S=4, f_{BW}=1, f_C=0.5 and Three Channels -1,0,+1 of 4-Path, 2-to-1 Down sample Analysis Filter Bank

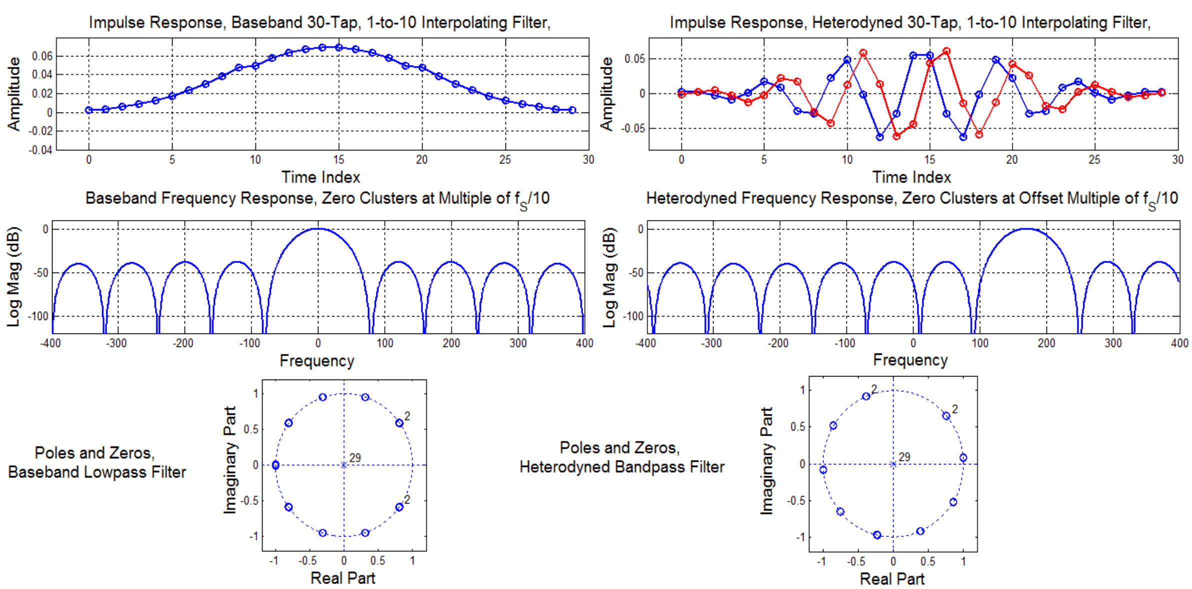

Figure 6.Impulse Response, Frequency Response and Pole-Zero Diagram of 1-to-10 Interpolator

its zeros on the remaining 9 spectral copies. We only compute the real output from this filter so that the output spectrum is Hermetian symmetric as shown in a later figure.

4 SIGNAL PROCESSING CHAIN

We now follow the transformations of the signal formed by the filter input stage through the two remaining stages. We will be able to see the function and performance of the successive filters in the spectral translate and interpolation processing chain. Figures 76a, 7b, and 7c show the input time series presented to the 80-path interpolator through the 4-path analysis filter at the sample rate 2 MHz and the output time series from the 80-path interpolator performing 1-to-40 up sampling to form the 80 MHz sample rate output. Also seen is the spectrum of the up-sampled channelizer output. Here we clearly see the channelizer's spectral suppression of the non-baseband input channel. The three figures differ in the center frequency of the output spectrum. Figure 7a corresponds to the up-sampled DC centered input baseband spectrum while figure 7b shows the up-sampled offset from DC by 0.5 MHz input spectrum introduced by the external heterodyne. Figure 7c shows the up-sampled and same 0.5 MHz offset from DC modified by the channelizer.

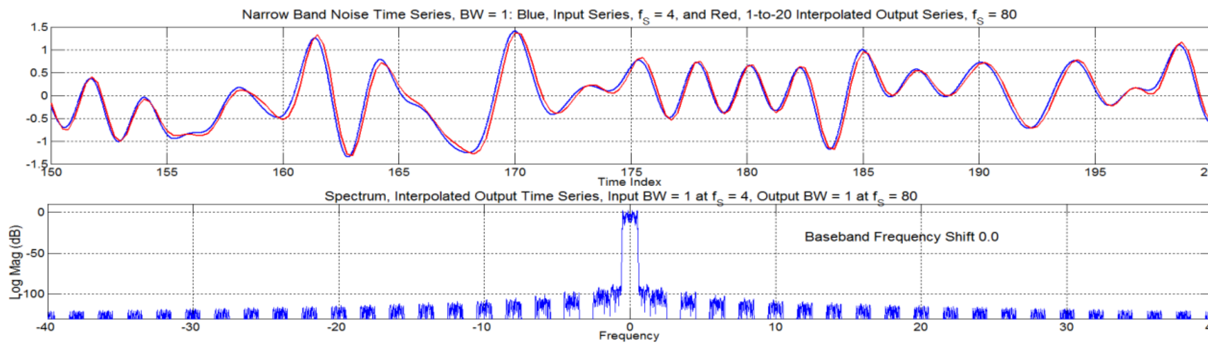

Figure 7a. Input and Output Time Series at 2-MHz and 80 MHz Sample Rates and Spectrum of Baseband Zero Frequency Input Signal Up Converted 1-to-40 Output Signal.

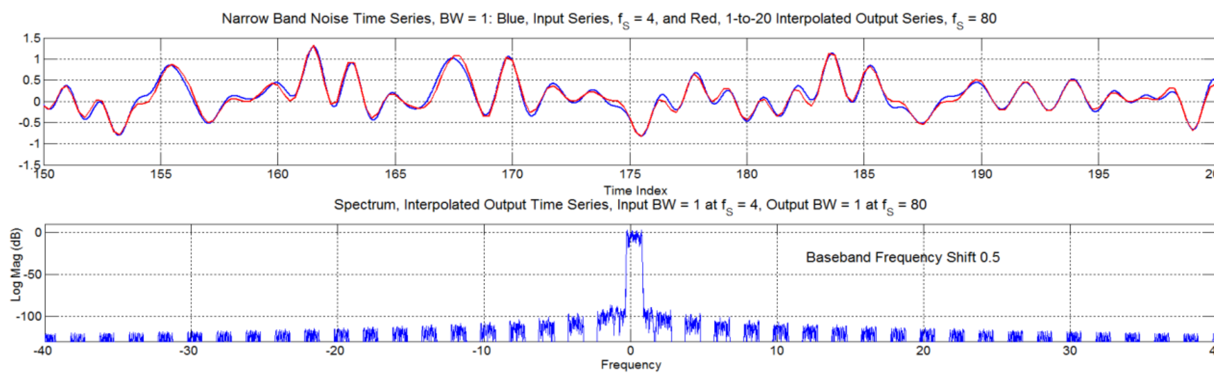

Figure 7b. Input and Output Time Series at 2-MHz and 80 MHz Sample Rates and Spectrum of 0.5 MHz Offset Frequency Input Signal Up Converted 1-to-40 Output Signal.

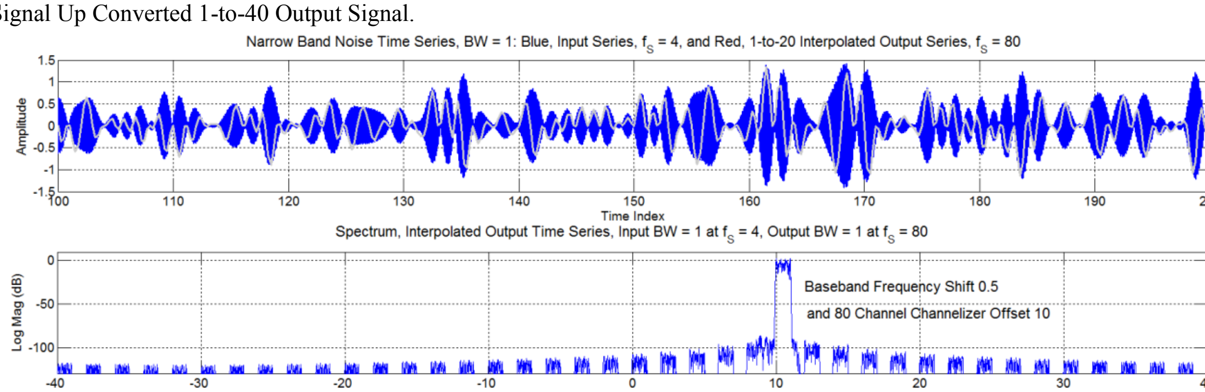

Figure 7c. Input and Output Time Series at 2-MHz and 80 MHz Sample Rates and Spectrum of 0.5 MHz Offset Frequency Input Signal Up Converted 1-to-40 Output Signal and Translated to 10 MHz in Synthesis Channelizer.

offset of 10 MHz by delivering the three input ports from the analysis filter to synthesizer bins 9, 10 and 11Figure 8 shows the spectrum of the 10-path 1-to-10 frequency offset interpolating filter along with its input and output spectrum. The filter increased the sample rate 1-to-10 and extracted the spectral copy of the input signal originally centered at 10.5 MHz from the second Nyquist zone which added 2 times the input sample rate of 80 MHz or 160 MHz to the Input signal to obtain the desired output at 170.5 MHz. The filter's clusters of 3-zeros at each 80 MHz offset from the desired spectral frequency suppressed the spectral replicas in the 9 other Nyquist zones by more than 100 dB. Figure 9 shows a section of the spectrum centered at 170.5 MHz from the lower subplot of figure 8.

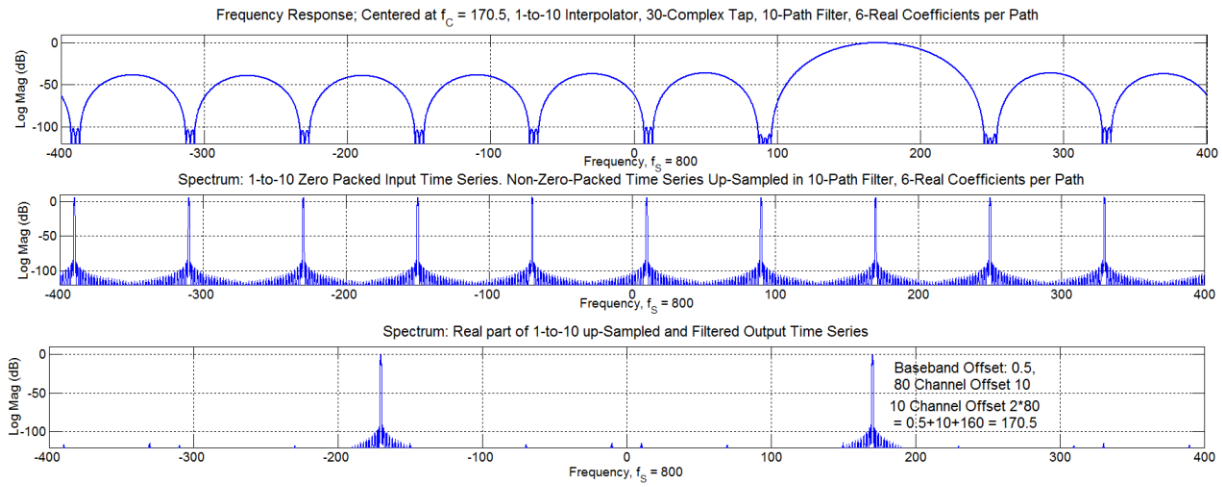

Figure 8. Spectral Response of Frequency Offset 10-Path 1-to-10 Interpolating Filter, Input Spectrum from the 80-Path 1-to-40 Interpolating and Frequency Shift Interpolating Filter and Output Spectrum from 10-path Filter

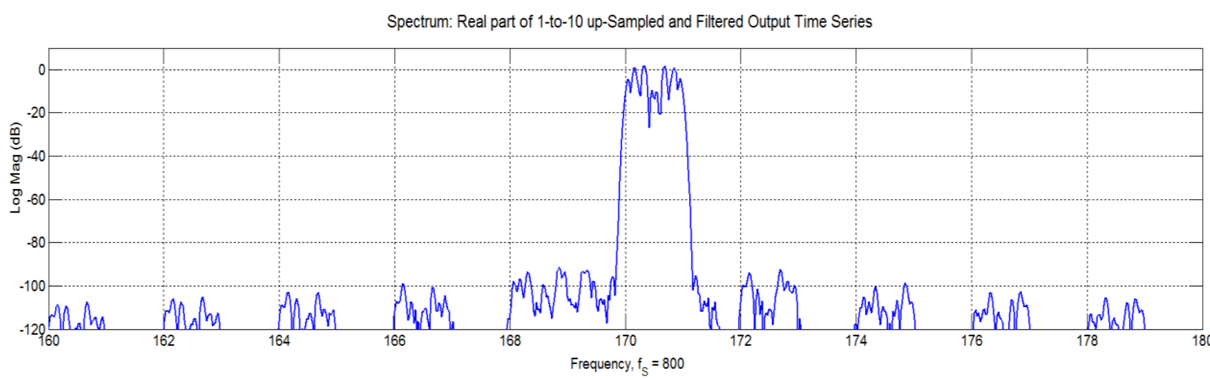

Figure 9. Zoom to Spectral Region of the Spectrum Centered at 170.5 MHz from the Lower Subplot of Figure 8.

4. CLOSING COMMENTS

The computational workload for the cascade of the three M-path filters proves to be less than 8-real multiplies per output sample. This is nearly half the workload of the single interpolator version we described as the second option and a 3600-to1 reduction relative to bandwidth reduction at the output sampling rate. The lesson learned here is always perform the signal processing tasks at the lowest sample rate satisfying the Nyquist criterion. We regularly use this perspective when down sampling and we should also be using the same perspective when up-sampling.

5. REFERENCES

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